Section 1.2 Objectives

- Recognize rational numbers.
- Simplify fractions.
- Add, subtract, multiply, and divide with fractions.
- Evaluate fractions raised to a power.
- Use order of operations to simplify arithmetic expressions that contain fractions.
INTRODUCTION

In this section, we will expand the set of numbers that we work with to include more than just the integers. We will work with a set of numbers called the rational numbers. You will learn to perform arithmetic operations with rational numbers. Put simply, the rational numbers involve fractions. So before we begin our formal study of the rational numbers, let’s review some basic vocabulary and concepts associated with fractions.

FRACTIONS

Fractions are used to indicate how many parts of a whole we have. Fractions contain two numbers separated by a fraction bar as shown below.

\[
\begin{align*}
\text{Numerator} & \leftarrow \text{how many equal parts we have} \\
\text{Denominator} & \leftarrow \text{how many equal parts make up a whole}
\end{align*}
\]

For example, if a pizza is cut into 8 equal pieces and you take 3 pieces, you have \(\frac{3}{8}\) of the pizza. The fraction is read “three eights” or “3 out of 8.”

Equivalent fractions have the same value. Equivalent fractions are obtained using one of the following methods.

- Multiply the numerator and denominator by the same number.
  
  \[
  \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}
  \]

- Divide the numerator and denominator by the same number.
  
  \[
  \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}
  \]

An improper fraction is written as a mixed number by dividing the denominator into the numerator.

**EXAMPLE:** Rewrite \(\frac{17}{5}\) as a mixed number.

\[
\frac{17}{5} = 3 \frac{2}{5}
\]

Whole number part \(\rightarrow 3\)

Denominator \(\rightarrow 5\)

\[
\frac{17}{15} \quad \Rightarrow \quad 3 \frac{2}{5}
\]

Numerator \(\rightarrow \frac{2}{5}\)

**PRACTICE:** Rewrite \(\frac{21}{8}\) as a mixed number.

Answer: \(2 \frac{5}{8}\)
Now we are ready to focus on the set of numbers called the rational numbers.

**RATIONAL NUMBERS**

An easy way to distinguish the rational numbers from the integers is that the *rational numbers* include fractions whereas the integers do not. Some examples of rational numbers are $\frac{3}{5}, -\frac{7}{6}$, and 0. Look at some other rational numbers on the number line below.

\[
-3 = \frac{-3}{1} \quad -1\frac{3}{4} = -\frac{7}{4} \quad \frac{3}{4} \quad 2\frac{1}{2} = \frac{5}{2}
\]

The mathematical definition of rational numbers is given below. The definition and the paragraphs that follow explain how to determine whether a number is a rational number.

<table>
<thead>
<tr>
<th>RATIONAL NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rational numbers</strong> are numbers that can be written as one of the following:</td>
</tr>
<tr>
<td>• a positive fraction</td>
</tr>
<tr>
<td>• a negative fraction</td>
</tr>
<tr>
<td>• zero</td>
</tr>
<tr>
<td><strong>NOTE:</strong> The numerator is any integer. The denominator is any integer except zero.</td>
</tr>
</tbody>
</table>

**Can a Rational Number Contain Zero?**

The definition above states that 0 is a rational number.

If a fraction contains 0 in the numerator, then the fraction is a rational number.

If a fraction contains 0 in the denominator, then the fraction is not a rational number.

<table>
<thead>
<tr>
<th>ZERO IN FRACTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{0}{n} = 0$</td>
</tr>
<tr>
<td>$\frac{n}{0} = \text{Undefined}$</td>
</tr>
</tbody>
</table>

**EXAMPLES:** Are the following numbers rational numbers?

1. $\frac{0}{4}$ Yes, this is a rational number since 0 is in the numerator.

2. $\frac{4}{0}$ No, this is not a rational number since 0 is in the denominator.
**ARE INTEGERS RATIONAL NUMBERS?** Yes, integers are rational numbers since they can be written as fractions with a denominator of 1.

**EXAMPLES:** Rewrite 3 and \(-12\) as fractions.

\[
3 = \frac{3}{1} \quad -12 = \frac{-12}{1}
\]

Write each integer as a fraction with a denominator of 1.

**Are Mixed Numbers Rational Numbers?** Yes, mixed numbers are rational numbers since they can be written as improper fractions.

**EXAMPLE:** Rewrite \(\frac{5}{8}\) as an improper fraction.

\[
1\frac{5}{8} = \frac{(1 \times 8) + 5}{8} = \frac{8 + 5}{8} = \frac{13}{8}
\]

Multiply the whole number with the denominator, and add that product to the numerator. Place the result in the numerator, and keep the same denominator.

**REVIEW:** CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION 🎤

**PRACTICE:** Rewrite the mixed number as an improper fraction.

1. \(3\frac{3}{4}\)  
2. \(6\frac{2}{5}\)

**Answers:**

1. \(\frac{15}{4}\) 🎤  
2. \(\frac{32}{5}\)

**Are Decimals Rational Numbers?** Decimals that terminate (end) and decimals that repeat a pattern are rational numbers since they can be written as fractions.

**EXAMPLES:** Rewrite the decimal as a fraction in simplest form.

1. \(23.571 = \frac{23571}{1000}\) Since the rightmost decimal place is the thousandths place, the denominator will be 1,000. (Hint: Since there are three digits to the right of the decimal point, there should be three zeros in the denominator, giving us the denominator 1,000.)

2. \(3.9 = \frac{39}{10}\) Since the rightmost decimal place is the tenths place, the denominator will be 10. (Hint: Since there is one digit to the right of the decimal point, there should be one zero in the denominator, giving us the denominator 10.)

**PRACTICE:** Rewrite each decimal as a fraction in simplest form.

1. \(39.201\)  
2. \(8.17\)

**Answers:**

1. \(\frac{39201}{1000}\) 🎤  
2. \(\frac{817}{100}\)
REVIEW: RATIONAL NUMBERS 🌱

EXAMPLE: Which of the following is NOT a rational number?

a. \( \frac{7}{3} \) This is a rational number because it is a positive fraction.

b. \( -\frac{2}{5} \) This is a rational number because it is a negative fraction.

c. \( \frac{0}{6} \) This is a rational number because only the numerator is 0.

d. \( \frac{8}{0} \) This is not a rational number because the denominator is 0.

e. 3 This is a rational number because the integer 3 can be written as the fraction \( \frac{3}{1} \).

f. \( 4\frac{2}{3} \) This is a rational number because the mixed number \( 4\frac{2}{3} \) can be written as the fraction \( \frac{14}{3} \).

g. 3.1 This is a rational number because the decimal 3.1 can be written as the fraction \( \frac{31}{10} \).

The answer is d.

PRACTICE: Which of the following is NOT a rational number? \( \frac{5}{6} \) \( -\frac{1}{4} \) 9 \( \frac{0}{2} \) \( \frac{3}{0} \) \( -8 \)

Answer: \( \frac{3}{0} \) is not a rational number because the denominator is 0.

PRACTICE: Which of the following is NOT a rational number? \( \frac{11}{8} \) \( -\frac{2}{0} \) \( 1\frac{3}{4} \) \( -\frac{2}{3} \) \( -4 \) 8.2 \( \frac{0}{7} \)

Answer: \( -\frac{2}{0} \) is not a rational number because the denominator is 0.

Simplifying Rational Numbers

An important skill necessary for working with rational numbers is simplifying fractions. In fact, if an answer to any math problem is a fraction, it is important to express the final answer as the fraction in simplified (reduced) form. The procedure for simplifying fractions is given below.

<table>
<thead>
<tr>
<th>SIMPLIFYING FRACTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>To simplify a fraction, divide the numerator and the denominator by the Greatest Common Factor (GCF). A common factor is a number that divides into both numbers without a remainder.</td>
</tr>
<tr>
<td>( \frac{a}{b} = \frac{a \div c}{b \div c} ) Note: ( b \neq 0, c \neq 0 )</td>
</tr>
</tbody>
</table>
CHAPTER 1 – Real Numbers and Algebraic Expressions

Section 1.2 – Rational Numbers

Remember that a fraction is a division problem. So, when you simplify a fraction, it is important to use the rules for dividing signed numbers.

<table>
<thead>
<tr>
<th>FRACTIONS AND NEGATIVES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{a}{b} = \frac{-a}{b} = -\frac{a}{b}) If only the numerator is negative or only the denominator is negative, the fraction is negative.</td>
</tr>
<tr>
<td>NOTE: Generally, we write the negative sign in the numerator or in front of the fraction. The negative sign is usually not written in the denominator because it is easy to miss it there.</td>
</tr>
<tr>
<td>(-\frac{a}{-b} = \frac{a}{b}) If both the numerator and the denominator are negative, the fraction is positive.</td>
</tr>
</tbody>
</table>

**EXAMPLES:** Simplify each fraction.

1. \(\frac{8}{28}\) The Greatest Common Factor (GCF) of 8 and 28 is 4.
   
   \[
   \frac{8}{28} = \frac{8 \div 4}{28 \div 4} = \frac{2}{7}
   \]
   
   This is the final answer in simplified form.

2. \(-\frac{15}{6}\) The Greatest Common Factor (GCF) of \(-15\) and 6 is 3.
   
   \[
   -\frac{15}{6} = -\frac{15 \div 3}{6 \div 3} = -\frac{5}{2}
   \]
   
   Rewrite the fraction with the negative sign in front of the fraction.
   
   \[
   -\frac{5}{2}
   \]
   
   This is the final answer in simplified form.

3. \(-\frac{18}{-10}\) The Greatest Common Factor (GCF) of \(-18\) and \(-10\) is 2.
   
   \[
   -\frac{18}{-10} = -\frac{18 \div 2}{-10 \div 2} = -\frac{9}{-5}
   \]
   
   A negative number divided by a negative number is a positive number.
   
   \[
   \frac{9}{5}
   \]
   
   This is the final answer in simplified form.
**PRACTICE**: Simplify.

1. \[\frac{24}{36}\]
2. \[-\frac{28}{12}\]
3. \[-\frac{70}{45}\]
4. \[\frac{0}{6}\]

**Answers**: 

1. \[\frac{2}{3}\]
2. \[-\frac{7}{3}\]
3. \[\frac{14}{9}\]
4. \[0\]

**ARITHMETIC OPERATIONS ON RATIONAL NUMBERS**

Now we will review the procedures for performing the four basic arithmetic operations on rational numbers: addition, subtraction, multiplication, and division.

**MULTIPLYING RATIONAL NUMBERS**

A common denominator is NOT needed to multiply fractions. Instead, we simplify if possible, then multiply straight across.

<table>
<thead>
<tr>
<th>MULTIPLYING RATIONAL NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Check to see if a numerator and a denominator have a common factor. If so, simplify by dividing out the GCF (Greatest Common Factor).</td>
</tr>
</tbody>
</table>

**EXAMPLES**: Perform the multiplication.

1. \[\left(\frac{-5}{7}\right) \left(\frac{3}{10}\right)\]  
   Check to see if a numerator and a denominator have a common factor. 
   \[= \left(\frac{-5}{7}\right) \left(\frac{3}{10}\right)\]  
   The GCF of \(-5\) and 10 is 5. 
   \[= \left(\frac{-1 \cdot 5}{7}\right) \left(\frac{3}{10 \cdot 2}\right)\]  
   Divide \(-5\) by 5. This gives \(-1\). Divide 10 by 5. This gives 2. 
   \[= \frac{-1 \cdot 3}{7 \cdot 2}\]  
   Multiply the remaining numerators together. Multiply the remaining denominators together. 
   \[= \frac{-3}{14}\]  
   Since the fraction cannot be simplified, this is the final answer.
2. \( \frac{-20}{9} \times \frac{-27}{8} \)  
Check to see if there are any common factors.

\[ = \frac{-20}{9} \times \frac{-27}{8} \]

The GCF of \(-20\) and \(8\) is \(4\).

\[ = \frac{-5 \times -20}{9 \times 2} \times \frac{-27}{8} \]

Divide \(-20\) by \(4\). This gives \(-5\).

Divide \(8\) by \(4\). This gives \(2\).

\[ = \frac{-5 \times -20}{9 \times 2} \times \frac{-27}{8} \]

The GCF of \(-27\) and \(9\) is \(9\).

\[ = \frac{-5 \times -3 \times -20}{1 \times 2 \times 9 \times 2} \times \frac{-27}{8} \]

Divide \(-27\) by \(9\). This gives \(-3\).

Divide \(9\) by \(9\). This gives \(1\).

\[ = \frac{-5 \times -3}{1 \times 2} \times \frac{-27}{8} \]

Multiply the remaining numerators together.

Multiply the remaining denominators together.

\[ = \frac{15}{2} \]

Since the fraction cannot be simplified, this is the final answer.

3. \( 0 \cdot \frac{2}{3} \)

0 times any number equals 0.

\[ = 0 \]

This is the final answer.

4. \( \frac{4}{3} \times 8 \)

Rewrite 8 as a fraction with a denominator of 1.

\[ = \frac{4}{3} \times \frac{8}{1} \]

There is no numerator / denominator pair with a GCF.

So, multiply the numerators together and multiply the denominators together.

\[ = \frac{32}{3} \]

Since the fraction cannot be simplified, this is the final answer.

**REVIEW: MULTIPLYING RATIONAL NUMBERS**

**PRACTICE:** Perform the multiplication.

1. \( \frac{5}{6} \times \frac{7}{9} \)

5. \( \frac{7}{9} \times 5 \)

9. \( \frac{3}{4} \times \frac{4}{16} \)

2. \( \left( \frac{2}{3} \right) \left( \frac{-2}{3} \right) \)

6. \( \frac{6}{7} \times \frac{2}{3} \)

10. \( \frac{4}{65} \times \frac{-13}{100} \)

3. \( \frac{4}{-11} \times \frac{-3}{7} \)

7. \( 5 \times \frac{4}{15} \)

11. \( \left( \frac{-4}{5} \right) \left( \frac{-25}{16} \right) \)

4. \( \frac{2}{13} \cdot 0 \)

8. \( \frac{3}{4} \times \frac{8}{15} \)

12. \( \frac{12}{15} \times \frac{10}{9} \)
Answers:

1. \[ \frac{35}{54} \]  
2. \[ -\frac{4}{9} \]  
3. \[ \frac{12}{77} \]  
4. 0  
5. \[ \frac{35}{9} \]  
6. \[ \frac{4}{7} \]  
7. \[ \frac{4}{3} \]  
8. \[ \frac{2}{5} \]  
9. \[ \frac{3}{16} \]  
10. \[ -\frac{1}{125} \]  
11. \[ \frac{5}{4} \]  
12. \[ \frac{8}{9} \]  

**DIVIDING RATIONAL NUMBERS**

A common denominator is NOT needed to divide fractions. We will divide fractions by performing a procedure similar to the procedure used to subtract integers. Recall how we changed a subtraction problem into an addition problem and added the opposite. Similarly, we will change a division problem into a multiplication problem and multiply by the reciprocal.

First you need to understand what is meant by a reciprocal. The **reciprocal** of a fraction is formed by switching the positions of the numerator (top) and the denominator (bottom). In other words, a reciprocal is a fraction flipped upside down!

The **reciprocal** of \( \frac{a}{b} \) is \( \frac{b}{a} \).

The **reciprocal** of \( a \) is \( \frac{1}{a} \).

**RECIPIROCAL OF A FRACTION**

| The reciprocal of \( \frac{a}{b} \) is \( \frac{b}{a} \). | The reciprocal of \( a \) is \( \frac{1}{a} \). |

**KEEP – CHANGE – FLIP METHOD**

Now you will learn how to change a division problem into a multiplication problem.

**KEEP:** keep the first fraction the same  
**CHANGE:** change the operation to multiplication  
**FLIP:** flip the second fraction to get its reciprocal

The division \( \frac{a}{b} \div \frac{c}{d} \) equals the multiplication \( \frac{a}{b} \cdot \frac{d}{c} \).

Note: \( b, c, \) and \( d \) cannot equal 0.
So, to divide fractions, you must multiply the first fraction by the reciprocal of the second fraction. Review the entire procedure for dividing fractions below.

**DIVIDING RATIONAL NUMBERS**

1. Change the division problem to a multiplication problem using\[ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \]
   KEEP – CHANGE – FLIP.

2. Simplify by dividing out common factors if possible.
   Note: Never do this before Step 1.

3. Multiply the numerators together. Multiply the denominators together.
   \[ = \frac{ad}{bc} \]

**EXAMPLE:** Perform the division.

\[ \frac{2}{5} \div \frac{2}{7} \]

Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP.

\[ \begin{array}{ccc}
\text{Keep} & \text{Change} & \text{Flip} \\
\downarrow & \downarrow & \downarrow \\
= \frac{2}{5} \times \frac{7}{2} & \text{The first fraction is being multiplied by the reciprocal of the second fraction.}
\end{array} \]

\[ = \frac{2 \times 1}{5 \times 7} \quad \text{Simplify by dividing out common factors. NEVER do this until the problem has been changed to multiplication.} \]

\[ = \frac{1 \times 7}{5 \times 1} \quad \text{Multiply the numerators together. Multiply the denominators together.} \]

\[ = \frac{7}{5} \quad \text{This is the final answer. It can be left as an improper fraction.} \]

**REVIEW:** DIVIDING RATIONAL NUMBERS 🌐

**PRACTICE:** Perform the division.

1. \[ \frac{-1}{3} \div \frac{4}{5} \]
2. \[ \frac{5}{7} \div \frac{10}{11} \]
3. \[ \frac{11}{24} \div \frac{55}{36} \]
4. \[ \frac{14}{3} \div \left( \frac{-21}{5} \right) \]
5. \[ \left( \frac{-7}{9} \right) \div \left( \frac{-7}{36} \right) \]
6. \[ \frac{15}{6} \div 20 \]
### Answers:

1. \(-\frac{5}{12}\)  
2. \(\frac{11}{14}\)  
3. \(\frac{3}{10}\)  
4. \(-\frac{10}{9}\)  
5. 4  
6. \(\frac{1}{8}\)

### Division With Zero

When you studied integers, you learned the rules for dividing when zero was in the problem. Those same rules apply to division problems that contain fractions. Review the rules below.

<table>
<thead>
<tr>
<th>Division Involving Zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \div n = 0)</td>
</tr>
<tr>
<td>If 0 is divided by any number (except 0), the <strong>answer is 0</strong>.</td>
</tr>
<tr>
<td>(n \div 0 = \text{undefined})</td>
</tr>
<tr>
<td>If any number is divided by 0, the <strong>answer is undefined</strong>.</td>
</tr>
<tr>
<td>In other words, there is no numerical answer.</td>
</tr>
</tbody>
</table>

#### EXAMPLES: Perform the division.

1. \(0 \div \frac{3}{5}\)  
   Zero divided by any number is 0.  
   \[= 0\]

2. \(\frac{2}{9} \div 0\)  
   Any number divided by 0 is undefined.  
   Undefined  
   Since the divisor was 0, the answer is undefined.

#### REVIEW: Division Involving Zero

#### PRACTICE: Perform the division.

1. \(0 \div \frac{4}{7}\)  
2. \(\frac{1}{2} \div 0\)

#### Answers:

1. 0  
2. Undefined
COMPLEX FRACTIONS

A complex fraction has one or two fractions within a fraction. While simplifying this type of problem may seem like it is going to be “complex,” it really is not. We will simply rewrite a complex fraction as the division of two fractions and then proceed as we did in the problems above.

COMPLEX FRACTIONS

<table>
<thead>
<tr>
<th>A complex fraction has a fraction in its numerator, denominator, or both.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The fraction to the right is a complex fraction. The numerator is the fraction $\frac{a}{b}$. The denominator is the fraction $\frac{c}{d}$. NOTE: $b$, $c$, and $d$ cannot equal 0.</td>
</tr>
<tr>
<td>2. Rewrite the complex fraction as the division of two fractions. (Work from the top down).</td>
</tr>
<tr>
<td>3. Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP.</td>
</tr>
<tr>
<td>4. Simplify by dividing out common factors if possible. Then multiply numerators together and denominators together.</td>
</tr>
</tbody>
</table>

**EXAMPLES:** Perform the division.

1. \[ \frac{\frac{2}{3}}{\frac{3}{8}} \]
   
   This is a complex fraction. Rewrite it as the division of two fractions. (Work from the top down).
   
   \[ = \frac{2}{3} \div \frac{3}{8} \]
   
   DO NOT divide out common factors while it is still a division problem.
   
   \[ \downarrow \downarrow \downarrow \]
   
   Change the division problem to a multiplication problem using KEEP – CHANGE – FLIP.
   
   \[ = \frac{2}{3} \times \frac{8}{3} \]
   
   There are no common factors to divide out. Multiply the numerators together. Multiply the denominators together.
   
   \[ = \frac{16}{9} \]
   
   This is the final answer.
2. \( \frac{-7}{3} \div \frac{14}{1} \)

This is a complex fraction. Rewrite it as the division of two fractions. 
(Work from the top down).

\[
= \frac{-7}{3} \div \frac{14}{1}
\]

We wrote the whole number 14 as \( \frac{14}{1} \).

\[
\downarrow \downarrow \downarrow \\
\text{K C F}
\]

Change the division problem to a multiplication problem using
KEEP – CHANGE – FLIP.

\[
= \frac{-7}{3} \times \frac{1}{14}
\]

Now that the problem has been changed to multiplication, we can look for common factors.

\[
= \frac{-1}{3} \times \frac{1}{2} \\
\]

Simplify by dividing out a common factor of 7.
Multiply the numerators together. Multiply the denominators together.

\[
= -\frac{1}{6}
\]

This is the final answer.

3. \( \frac{16}{6} \div \frac{5}{3} \)

This is a complex fraction. Rewrite it as the division of two fractions. 
(Work from the top down).

\[
= \frac{16}{1} \div \frac{6}{5}
\]

We wrote the whole number 16 as \( \frac{16}{1} \).

\[
\downarrow \downarrow \downarrow \\
\text{K C F}
\]

Change the division problem to a multiplication problem using
KEEP – CHANGE – FLIP.

\[
= \frac{16}{1} \times \frac{5}{6}
\]

Now that the problem has been changed to multiplication, we can look for common factors.

\[
= \frac{8}{1} \times \frac{5}{3}
\]

Simplify by dividing out a common factor of 2.
Multiply the numerators together. Multiply the denominators together.

\[
= \frac{40}{3}
\]

This is the final answer.

**PRACTICE:** Perform the division.

1. \( \frac{8}{19} \div \frac{4}{19} \)
2. \( \frac{-4}{5} \div \frac{-3}{10} \)
3. \( \frac{8}{5} \div \frac{4}{5} \)
4. \( \frac{-2}{5} \div \frac{7}{5} \)

**Answers:**

1. 2
2. \( \frac{8}{3} \)
3. \( \frac{2}{5} \)
4. \( \frac{-14}{5} \)
EXPONENTS AND FRACTIONS

In the previous section, exponents were used with integers. Now, exponents will be used with fractions. If a fraction is raised to a power, the fraction is placed in parentheses and the exponent is written on the outside. The fraction in parentheses is called the base. Recall that the exponent expresses a repeated multiplication of the base. More specifically, the exponent specifies how many times the base is used in the multiplication.

EXAMPLES: Evaluate.

1. \( \left( \frac{2}{5} \right)^3 \)

   The base is the fraction \( \frac{2}{5} \) inside the parentheses, and the exponent is 3.

   \[
   \left( \frac{2}{5} \right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = \frac{8}{125}
   \]

   This is the answer.

2. \( \left( -\frac{3}{4} \right)^2 \)

   The base is the fraction \( -\frac{3}{4} \) inside the parentheses, and the exponent is 2.

   \[
   \left( -\frac{3}{4} \right)^2 = \left( -\frac{3}{4} \right) \cdot \left( -\frac{3}{4} \right) = \frac{-3 \cdot -3}{4 \cdot 4} = \frac{9}{16}
   \]

   This is the answer.

PRACTICE: Evaluate.

1. \( \left( \frac{3}{4} \right)^3 \)

2. \( \left( -\frac{2}{3} \right)^2 \)

3. \( \left( \frac{6}{5} \right)^2 \)

4. \( \left( -\frac{3}{5} \right)^3 \)

Answers:

1. \( \frac{27}{64} \)

2. \( \frac{4}{9} \)

3. \( \frac{36}{25} \)

4. \( -\frac{27}{125} \)
**Adding and Subtracting Rational Numbers**

It is important to remember that two fractions must have the same denominator in order to add or subtract them. If the denominators are not the same, you will need to find a common denominator.

A common denominator (also called common multiple) is a number that both of the denominators can divide into without a remainder. You can find a common denominator of two numbers by multiplying them together. However, when adding or subtracting fractions it is best to use the **LCD - Least Common Denominator** (also called **LCM – Least Common Multiple**). A procedure for finding the **Least Common Denominator** is given below.

**Least Common Denominator (LCD)**

To determine the LCD, multiply the larger denominator by 1, by 2, by 3, and so on until one of the answers is divisible by the other denominator with no remainder.

**Example:** Determine the LCD needed to add \( \frac{2}{3} + \frac{4}{5} \).

The larger denominator is 5.

\[
\begin{align*}
5 \times 1 &= 5 & \text{5 is not evenly divisible by 3 (the other denominator)} \\
5 \times 2 &= 10 & \text{10 is not evenly divisible by 3 (the other denominator)} \\
5 \times 3 &= 15 & \text{15 is evenly divisible by 3 (the other denominator)} \\
\text{LCD} &= 15 & \text{Therefore, the LCD is 15.}
\end{align*}
\]

So, if you are adding or subtracting two fractions that do not have the same denominator, you will begin by finding the LCD. Then you will rewrite the fractions as equivalent fractions with the LCD. Once the two fractions have the same denominator, you simply add or subtract the fractions by combining the numerators.

**Adding and Subtracting Rational Numbers**

1. If the fractions have the same denominator, add the **numerators** by following the rules for adding and subtracting integers. Keep the denominator the same.

2. If the fractions do not have the same denominator, rewrite the fractions with the Least Common Denominator (LCD). Then follow the procedure in Step 1 above.

* To add or subtract rational numbers, the fractions must have the same denominator.

* Remember to simplify your final answer if possible.
EXAMPLES: Perform the indicated operation.

1. \( \frac{2}{3} + \frac{4}{5} \)

   The fractions cannot be added because they do not have the same denominator. We must find the LCD, a number that both denominators will divide into without a remainder. The LCD for 3 and 5 is 15. (Work shown in Example on last page.)

   Rewrite each fraction so the denominator will be 15 (the LCD).

   \[
   = \frac{2 \cdot 5}{3 \cdot 5} + \frac{4 \cdot 3}{5 \cdot 3}
   \]

   \[
   = \frac{10}{15} + \frac{12}{15}
   \]

   The fractions have a common denominator now.

   \[
   = \frac{10 + 12}{15}
   \]

   Add the numerators. Keep the denominator the same.

   \[
   = \frac{22}{15}
   \]

   This fraction cannot be simplified. This is the final answer.

2. \( \frac{-5}{6} - \frac{1}{2} \)

   The fractions cannot be subtracted because they do not have the same denominator. We must find the LCD, a number that both denominators will divide into evenly. The LCD for 6 and 2 is 6. (Use the procedure on the previous page to find the LCD.)

   Rewrite each fraction so the denominator will be 6 (the LCD).

   \[
   = \frac{-5}{6} - \frac{1 \cdot 3}{2 \cdot 3}
   \]

   \[
   = \frac{-5 - 3}{6}
   \]

   The fractions have a common denominator now.

   \[
   = \frac{-5 - 3}{6}
   \]

   Combine the numerators. Keep the denominator the same.

   \[
   = \frac{-8}{6}
   \]

   This fraction can be simplified because the numerator and denominator have a common factor of 2.

   \[
   = \frac{-8 \div 2}{6 \div 2}
   \]

   \[
   = \frac{-4}{3}
   \]

   Move the negative sign in front of the fraction.

   \[
   = -\frac{4}{3}
   \]

   This is the final answer.
3. \[
\frac{3}{8} - \frac{5}{6}
\]
In order to subtract the fractions, we must get a common denominator. The LCD for 8 and 6 is 24. (Use the procedure you learned earlier to find the LCD.)

Rewrite each fraction so the denominator will be 24 (the LCD).

\[
\frac{3 \cdot 3}{8 \cdot 3} - \frac{5 \cdot 4}{6 \cdot 4}
\]
1st Fraction: Multiply the denominator by 3 in order to get 24 (the LCD).
Multiply the numerator by 3 also.

2nd Fraction: Multiply the denominator by 4 in order to get 24 (the LCD).
Multiply the numerator by 4 also.

\[
= \frac{9}{24} - \frac{20}{24}
\]
The fractions have a common denominator now.

\[
= \frac{9 - 20}{24}
\]
Combine the numerators. Keep the denominator the same.

\[
= \frac{9 + (-20)}{24}
\]
In the numerator, change the subtraction to adding the opposite. Then add.

\[
= \frac{-11}{24}
\]
This fraction cannot be simplified. We will just move the negative sign in front of the fraction.

\[
= \frac{-11}{24}
\]
This is the final answer.

4. \[
5 - \frac{1}{2}
\]
Rewrite the whole number 5 a a fraction with a denominator of 1.

\[
= \frac{5}{1} - \frac{1}{2}
\]
In order to subtract the fractions, we must get a common denominator. The LCD for 1 and 2 is 2.

Rewrite each fraction so the denominator will be 2 (the LCD).

\[
= \frac{5 \times 2}{1 \times 2} - \frac{1}{2}
\]
1st Fraction: Multiply the denominator by 2 in order to get 2 (the LCD).
Multiply the numerator by 2 also.

2nd Fraction: The denominator is already 2.

\[
= \frac{10}{2} - \frac{1}{2}
\]
The fractions have a common denominator now.

\[
= \frac{10 - 1}{2}
\]
Subtract the numerators. Keep the denominator the same.

\[
= \frac{9}{2}
\]
This fraction cannot be simplified. This is the final answer as an improper fraction.
REVIEW: ADDING RATIONAL NUMBERS (LIKE DENOMINATORS)

REVIEW: ADDING RATIONAL NUMBERS (UNLIKE DENOMINATORS)

PRACTICE: Perform the indicated operation.

1. \( \frac{-1}{6} + \frac{-2}{6} \)
2. \( \frac{3}{5} + \frac{1}{10} \)
3. \( \frac{1}{6} + \frac{5}{9} \)
4. \( \frac{-1}{6} + \frac{3}{4} \)
5. \( \frac{5}{9} + \frac{11}{12} \)
6. \( 6 - \frac{2}{3} \)
7. \( \frac{3}{5} - \frac{1}{10} \)
8. \( \frac{1}{4} - \frac{7}{10} \)
9. \( -\frac{3}{11} - \frac{1}{3} \)
10. \( -\frac{8}{9} - 4 \)
11. \( \frac{1}{9} - \frac{4}{3} \)
12. \( 2 + \frac{7}{8} \)
13. \( \frac{5}{12} + \frac{1}{3} \)
14. \( \frac{4}{3} + \left( -\frac{9}{5} \right) \)
15. \( \frac{2}{3} - \frac{1}{6} \)
16. \( -\frac{4}{5} - \frac{3}{7} \)

Answers:

1. \(-\frac{1}{2} \)
2. \(\frac{7}{10} \)
3. \(\frac{13}{18} \)
4. \(\frac{7}{12} \)
5. \(\frac{53}{36} \)
6. \(\frac{16}{3} \)
7. \(\frac{1}{2} \)
8. \(-\frac{9}{20} \)
9. \(-\frac{20}{33} \)
10. \(-\frac{44}{9} \)
11. \(-\frac{11}{9} \)
12. \(\frac{23}{8} \)
13. \(\frac{3}{4} \)
14. \(-\frac{7}{15} \)
15. \(\frac{1}{2} \)
16. \(-\frac{43}{35} \)
ORDER OF OPERATIONS WITH RATIONAL NUMBERS

Now that you have studied how to add, subtract, multiply, and divide fractions, we can evaluate more complicated expressions where fractions are involved. These expressions will involve more than one operation. The operations must be performed in the same order that you learned when you studied integers. Review the *Order of Operation* rules listed below.

<table>
<thead>
<tr>
<th>ORDER OF OPERATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1: Parentheses</strong></td>
</tr>
<tr>
<td>If there are any operations in parentheses, those computations should be performed first.</td>
</tr>
<tr>
<td><strong>Step 2: Exponents and Roots</strong></td>
</tr>
<tr>
<td>Simplify any numbers being raised to a power and any numbers under the $\sqrt{}$ symbol.</td>
</tr>
<tr>
<td><strong>Step 3: Multiplication and Division</strong></td>
</tr>
<tr>
<td>Do these two operations in the order in which they appear, working from <em>left to right</em>.</td>
</tr>
<tr>
<td><strong>Step 4: Addition and Subtraction</strong></td>
</tr>
<tr>
<td>Do these two operations in the order in which they appear, working from <em>left to right</em>.</td>
</tr>
</tbody>
</table>

**EXAMPLES**: Evaluate.

1. $\frac{2}{3} + \frac{3}{4} \times \frac{1}{2}$

   **Step 1: Parentheses** – There are none.

   **Step 2: Exponents and Roots** – There are none.

   $= \frac{2}{3} + \frac{3}{4} \times \frac{1}{2}$

   **Step 3: Multiplication and Division**

   Since there are no common factors to divide out, just multiply straight across.

   $= \frac{2}{3} + \frac{3}{8}$

   **Step 4: Addition and Subtraction**

   To add these fractions, we need a common denominator.

   The LCD for 3 and 8 is 24.

   $= \frac{2 \times 8}{3 \times 8} + \frac{3 \times 3}{8 \times 3}$

   To get 24 as the common denominator:

   - Multiply the numerator and denominator of the first fraction by 8.
   - Multiply the numerator and denominator of the second fraction by 3.

   $= \frac{16}{24} + \frac{9}{24}$

   The denominators are the same now.

   Add the numerators and keep the same denominator.

   $= \frac{25}{24}$

   **Answer**: This fraction cannot be simplified.

   This is the final answer.
2. \[ \frac{6}{7} \div \frac{11}{14} \times \frac{3}{5} \]

**Step 1: Parentheses** – There are none.

**Step 2: Exponents and Roots** – There are none.

\[ \frac{6}{7} \div \frac{11}{14} \times \frac{3}{5} \]

**Step 3: Multiplication and Division**

- There is a multiplication and a division.
- We will do the division first since it is the leftmost operation.

\[ \frac{6}{7} \times \frac{14}{11} \times \frac{3}{5} \]

To change the division problem to a multiplication problem, multiply the 1\textsuperscript{st} fraction by the reciprocal of the 2\textsuperscript{nd} fraction (*Keep-Change-Flip*).

\[ \frac{6}{1} \times \frac{2}{11} \times \frac{3}{5} \]

- Simplify by dividing 7 and 14 by the common factor 7.

\[ \frac{6 \times 2}{1 \times 11} \times \frac{3}{5} \]

- Multiply the first two fractions together.

\[ \frac{12}{11} \times \frac{3}{5} \]

- Multiply the result by the third fraction.
- Since there are no common factors to divide out, just multiply straight across.

\[ \frac{36}{55} \]

**Answer:** This fraction cannot be simplified.
- This is the final answer.

3. \[ \frac{4}{7} \times \left( \frac{2}{3} - \frac{1}{9} \right) \]

**Step 1: Parentheses** – Begin with the operation inside the parentheses.

- To subtract the fractions, we need a common denominator.
- The LCD for 3 and 9 is 9.

\[ \frac{4}{7} \times \left( \frac{2 \times 3}{3 \times 3} - \frac{1}{9} \right) \]

- To get 9 as the common denominator:
  - Multiply the numerator and denominator of the first fraction in the parentheses by 3.
  - The second fraction in the parentheses does not need to be changed.

\[ \frac{4}{7} \times \left( \frac{6}{9} - \frac{1}{9} \right) \]

- The fractions in the parentheses have the same denominator now.
- Subtract the numerators and keep the same denominator.

**Step 2: Exponents and Roots** – There are none.

\[ \frac{4}{7} \times \frac{5}{9} \]

**Step 3: Multiplication and Division**

- Since there are no common factors to divide out, just multiply straight across.

\[ \frac{20}{63} \]

**Answer:** This fraction cannot be simplified.
- This is the final answer.
4. \( 3\left(\frac{1}{4}\right)^2 - 5^0 \)

\[ \textbf{Step 1: Parentheses} \quad \text{There is nothing to simplify inside the parentheses.} \]

\[ = 3\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) - 5^0 \quad \frac{1}{4} \text{ two times. } \]

\[ \text{Recall that any number (other than 0) raised to the 0 power is 1. } \]

\[ = 3\left(\frac{1}{16}\right) - 1 \quad \textbf{Step 3: Multiplication and Division} \]

\[ \text{Express 3 as a fraction in order to multiply.} \]

\[ = \frac{3}{16} \left(\frac{1}{16}\right) - 1 \quad \text{Since there are no common factors to divide out, just multiply straight across.} \]

\[ = \frac{3}{16} - 1 \quad \textbf{Step 4: Addition and Subtraction} \]

\[ \text{Write 1 as a fraction.} \]

\[ = \frac{3}{16} - \frac{1}{1} \quad \text{To subtract the fractions, we need a common denominator.} \]

\[ \text{The LCD for 16 and 1 is 16.} \]

\[ = \frac{3}{16} - \frac{1 \times 16}{1 \times 16} \quad \text{To get 16 as the common denominator:} \]

\[ \bullet \text{ Leave the first fraction the same.} \]

\[ \bullet \text{ Multiply the numerator and denominator of the second fraction by 16.} \]

\[ = \frac{3}{16} - \frac{16}{16} \quad \text{Combine the numerators.} \]

\[ \text{Keep the denominator the same.} \]

\[ = \frac{3 - 16}{16} \quad \text{Change the subtraction to adding the opposite.} \]

\[ = \frac{3 + (-16)}{16} \quad \text{Simplify the numerator.} \]

\[ = \frac{-13}{16} \quad \textbf{Answer: } \text{This fraction cannot be simplified.} \]

\[ \text{We will rewrite the answer with the negative sign in front of the fraction.} \]

\[ = \frac{-13}{16} \quad \text{This is the final answer.} \]
5. \[
\frac{5(3-6) + 3 \cdot 4}{2^3 - 12}
\]

In this problem, the fraction bar acts as parentheses, grouping the numerator together and grouping the denominator together.

First, simplify the numerator.

\[
= \frac{5(3-6) + 3 \cdot 4}{2^3 - 12}
\]

**Step 1: Parentheses**
Change the subtraction in the parentheses to adding the opposite.

\[
= \frac{5(3+(-6)) + 3 \cdot 4}{2^3 - 12}
\]

**Step 2: Exponents and Roots** – There are none in the numerator.

\[
= \frac{5(-3) + 3 \cdot 4}{2^3 - 12}
\]

**Step 3: Multiplication and Division** – Perform the two multiplications in the numerator.

\[
= \frac{-15 + 12}{2^3 - 12}
\]

**Step 4: Addition and Subtraction** – Add the integers in the numerator.

\[
= \frac{-3}{2^3 - 12}
\]

This is the simplified numerator.

Next, simplify the denominator.

\[
= \frac{-3}{2^3 - 12}
\]

**Step 1: Parentheses** – There are none in the denominator.

\[
= \frac{-3}{2 \times 2 \times 2 - 12}
\]

**Step 2: Exponents and Roots** – To evaluate \(2^3\), multiply 2 three times.

\[
= \frac{-3}{8 - 12}
\]

**Step 3: Multiplication and Division** – There are none in the denominator.

**Step 4: Addition and Subtraction**

In the denominator, change the subtraction to adding the opposite.

Add the integers in the denominator.

\[
= \frac{-3}{8 + (-12)}
\]

Simplify the fraction. A negative divided by a negative is a positive.

\[
= \frac{-3}{-4}
\]

\[
= \frac{3}{4}
\]

**Answer:** This is the final answer.
PRACTICE: Evaluate.

1. \(\frac{1}{2} + \frac{3}{5} \times \frac{5}{7}\)
2. \(\frac{5}{8} \div \frac{9}{10} \times \frac{3}{7}\)
3. \(\frac{3}{8} \times \left(\frac{5}{6} - \frac{1}{4}\right)\)
4. \(\left(\frac{-2}{9} + \frac{-1}{6}\right) \div 14\)
5. \(\left(\frac{3}{4}\right)^2 - \left(\frac{7}{8}\right)^0\)
6. \(\frac{9}{4} \left(\frac{-2}{3}\right) + \left(\frac{1}{2}\right)^3\)
7. \(\frac{12 + 2^3}{3 \cdot 7 - (4 - 2)}\)
8. \(\frac{(-9 + 5) \div 4 - 2}{(-4)^2 - 6}\)

Answers:

1. \(\frac{13}{14}\)
2. \(\frac{25}{84}\)
3. \(\frac{7}{32}\)
4. \(\frac{1}{36}\)
5. \(\frac{19}{8}\)
6. \(-\frac{11}{8}\)
7. \(\frac{20}{19}\)
8. \(-\frac{3}{10}\)
# SECTION 1.2 SUMMARY

## Rational Numbers

<table>
<thead>
<tr>
<th>Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Numerator</strong> → # of equal parts used</td>
</tr>
<tr>
<td><strong>Example</strong>:</td>
</tr>
</tbody>
</table>

| Zero in Fractions: | \( \frac{0}{n} = 0 \) | \( \frac{n}{0} \) Undefined |
| **Examples**: | \( \frac{0}{2} = 0 \) | \( \frac{2}{0} \) Undefined |

| Write a Mixed Number as an Improper Fraction |
| **Example**: | \( 6\frac{2}{7} = \frac{(6 \times 7) + 2}{7} = \frac{42 + 2}{7} = \frac{44}{7} \) |

| Write a Decimal as a Fraction |
| **Example**: | \( 26.31 = \frac{2631}{100} \) |

<table>
<thead>
<tr>
<th>Rational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>A rational number is any number that can be written as a fraction in the form:</td>
</tr>
<tr>
<td><strong>Numerator</strong> → any integer</td>
</tr>
<tr>
<td><strong>Denominator</strong> → any integer except 0</td>
</tr>
<tr>
<td><strong>Example</strong>: Which of the following is NOT a rational number?</td>
</tr>
<tr>
<td>[ \begin{align*} 1\frac{2}{3} &amp; \quad \frac{-7}{9} &amp; \quad 5.1 &amp; \quad \frac{0}{3} &amp; \quad \frac{2}{0} &amp; \quad -6 \ \end{align*} ]</td>
</tr>
<tr>
<td>( \frac{2}{0} ) is not a rational number because the denominator is 0.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simplifying Rational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>GCF – divide the numerator and denominator by the Greatest Common Factor</td>
</tr>
<tr>
<td><strong>Example</strong>:</td>
</tr>
</tbody>
</table>

| Negatives: |
| \( \frac{-a}{b} = \frac{a}{b} \) |
| **Examples**: | \( \frac{-5}{7} = \frac{5}{7} \) |
| \( \frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b} \) |
| **Examples**: | \( \frac{-5}{7} = \frac{5}{-7} = -\frac{5}{7} \) |

<table>
<thead>
<tr>
<th>Multiplying Rational Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example</strong>:</td>
</tr>
</tbody>
</table>

- 1. Divide out common factors in the numerator and denominator.
- 3. Simplify if possible.
### Dividing Rational Numbers

Change the division problem to a multiplication problem using **KEEP – CHANGE – FLIP.**

$$\frac{5}{6} \div \frac{2}{7}$$

Multiply the first fraction by the reciprocal of the second fraction.

$$\frac{5}{6} \times \frac{7}{2} = \frac{35}{12}$$

**Examples:**

- Division with Zero: $0 \div n = 0$
- $n \div 0$ is undefined

**Examples:**

- $0 \div \frac{8}{9} = 0$
- $\frac{8}{9} \div 0$ is undefined

**Division with Complex Fractions**

1. Rewrite the problem as the division of two fractions.
2. Change to a multiplication problem using **KEEP – CHANGE – FLIP.**

**Example:**

$$\frac{3}{5} \div \frac{4}{9} = \frac{3}{5} \times \frac{9}{4} = \frac{27}{20}$$

### Exponents

The exponent specifies how many times to use the base in a repeated multiplication.

**Example:**

$$\left(\frac{8}{9}\right)^2 = \left(\frac{8}{9} \times \frac{8}{9}\right) = \frac{8 \times 8}{9 \times 9} = \frac{64}{81}$$

### Adding and Subtracting Rational Numbers

1. Determine the LCD.
2. Rewrite the fractions by multiplying the numerator and denominator of each fraction by a number to obtain the LCD.
3. Add or subtract the numerators as indicated and keep the same denominator.
4. Simplify if possible.

**Example:**

$$\frac{1}{4} + \frac{5}{6} \quad \text{LCD} = 12$$

$$\frac{1 \times 3}{4 \times 3} + \frac{5 \times 2}{6 \times 2} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12}$$

### Order of Operations

1. Parentheses
2. Exponents and Roots
3. Multiplication and Division (work from left to right whichever operation comes first)
4. Addition and Subtraction (work from left to right whichever operation comes first)
SECTION 1.2 EXERCISES

Rational Numbers

Write each number as indicated.

1. Write \( \frac{7}{9} \) as an improper fraction.
2. Write 11.23 as a fraction.

Which of the following is NOT a rational number.

3. \(-5\), \(0\), \(6.3\), \(-\frac{5}{3}\), \(-\frac{1}{5}\), \(\frac{1}{5}\), \(2\), \(-\frac{2}{7}\)

Simplify.

4. \(\frac{30}{48}\)
5. \(\frac{120}{80}\)
6. \(-\frac{12}{40}\)
7. \(-\frac{15}{35}\)
8. \(\frac{15}{0}\)
9. \(0\)

Perform the indicated operation.

10. \(\frac{14 \times 8}{12 \times 21}\)
11. \(\frac{2}{3} \times 15\)
12. \(\frac{4}{5} \times -\frac{6}{7}\)
13. \(\left(\frac{-3}{8}\right) \left(\frac{5}{12}\right)\)
14. \(\frac{-15 \times 8}{2 \times -25}\)
15. \(0 \times \frac{11}{43}\)

16. \(\frac{7}{12} \div \frac{7}{4}\)
17. \(\frac{3}{2} \div 24\)
18. \(\frac{44}{15} \div -\frac{11}{9}\)
19. \(-20 \div \frac{4}{5}\)
20. \(\left(\frac{-5}{8}\right) \div \left(-\frac{30}{12}\right)\)
21. \(-\frac{5}{9} \div 0\)
22. \(0 \div \frac{6}{11}\)
23. \(\frac{\frac{2}{3}}{\frac{4}{5}}\)
24. \(\frac{-5}{-\frac{2}{3}}\)
25. \(\frac{7}{-14}\)

Evaluate.

26. \(\left(\frac{3}{7}\right)^2\)
27. \(\left(-\frac{2}{5}\right)^2\)
28. \(\left(\frac{2}{3}\right)^3\)
29. \(\left(-\frac{1}{2}\right)^3\)
Perform the indicated operation.

30. \(-\frac{3}{14} + \left(-\frac{5}{14}\right)\)

31. \(\frac{3}{8} + \frac{1}{7}\)

32. \(9 + \frac{3}{4}\)

33. \(-\frac{2}{5} + \frac{-4}{15}\)

34. \(-\frac{3}{4} + \frac{5}{6}\)

35. \(\frac{5}{8} + \left(-\frac{11}{12}\right)\)

36. \(\frac{1}{4} - \frac{3}{5}\)

37. \(-\frac{3}{4} - \frac{-2}{3}\)

38. \(\frac{3}{8} - \frac{5}{12}\)

39. \(-7 - \frac{2}{3}\)

40. \(\frac{4}{9} - 6\)

Evaluate each expression.

41. \(\frac{5}{6} \div \left(\frac{2}{3}\right)^2\)

42. \(\frac{1}{3} \div 5 \times \frac{1}{2}\)

43. \(\left(-\frac{2}{3}\right)^2 - 4\)

44. \(\frac{3}{8} - \frac{5}{6} \times \frac{12}{7}\)

45. \(\left(\frac{4}{15} + \frac{-5}{3}\right) - \left(\frac{2}{5}\right)^2\)

46. \(9 + \frac{3}{4} \div 8\)

47. \(2 - \frac{1}{3} \times \frac{1}{2}\)

48. \(\left(\frac{2}{9} - \frac{4}{9}\right) \div 3\)

49. \(\frac{14 + 3^3}{4 \cdot 5 - (6-10)}\)

50. \(\frac{2(-9+7) + 8 \cdot 4}{5^2 - 3}\)
### Answers to Section 1.2 Exercises

1. \( \frac{25}{9} \)  
2. \( \frac{1123}{100} \)  
3. \( \frac{2}{0} \)  
4. \( \frac{5}{8} \)  
5. \( \frac{3}{2} \)  
6. \( -\frac{3}{10} \)  
7. \( \frac{3}{7} \)  
8. Undefined  
9. 0  
10. \( \frac{4}{9} \)  
11. 10  
12. \( -\frac{24}{35} \)  
13. \( -\frac{5}{32} \)  
14. \( \frac{12}{5} \)  
15. 0  
16. \( \frac{1}{3} \)  
17. \( \frac{1}{16} \)  
18. \( -\frac{12}{5} \)  
19. -25  
20. \( \frac{1}{4} \)  
21. Undefined  
22. 0  
23. \( \frac{5}{6} \)  
24. \( \frac{15}{2} \)  
25. \( -\frac{1}{4} \)  
26. \( \frac{9}{49} \)  
27. \( \frac{4}{25} \)  
28. \( \frac{8}{27} \)  
29. \( -\frac{1}{8} \)
30. $\frac{-4}{7}$
31. $\frac{29}{56}$
32. $\frac{39}{4}$
33. $\frac{-2}{3}$
34. $\frac{1}{12}$
35. $\frac{-7}{24}$
36. $\frac{-7}{20}$
37. $\frac{-17}{12}$
38. $\frac{-1}{24}$
39. $\frac{-23}{3}$
40. $\frac{-50}{9}$
41. $\frac{15}{8}$
42. $\frac{1}{5}$
43. $\frac{-32}{9}$
44. $\frac{-59}{56}$
45. $\frac{-39}{25}$
46. $\frac{291}{32}$
47. $\frac{11}{6}$
48. $\frac{-2}{27}$
49. $\frac{41}{24}$
50. $\frac{14}{11}$
Mixed Review

Section 1.1 – 1.2

Simplify.

1. \[-8\]
2. \(-7 + 5\)
3. \(-4 - 6\)
4. \(-8 - (-9)\)
5. \(-7 \cdot (-8)\)
6. \(80 ÷ (-4)\)
7. \(6 ÷ 0\)
8. \(5^3\)
9. \((-9)^2\)
10. \(\sqrt[3]{8}\)
11. \(30 - 6^2 + \frac{1}{3}\)
12. \((15 + -3) ÷ \sqrt{16} \times 8\)
13. \(-3 - 7 + 5 \times (-6 + 2)\)
14. \(-3^2 + 4 \left(\frac{1}{3} ÷ \frac{1}{9}\right) - \frac{2}{3}\)
15. \(-6 ÷ \left(\frac{11}{4} - \frac{3}{4}\right) + \sqrt{25} \times \frac{2}{5}\)

Answers to Mixed Review

1. \(-8\)
2. \(-2\)
3. \(-10\)
4. \(1\)
5. \(56\)
6. \(-20\)
7. Undefined
8. 125
9. 81
10. 2
11. \(-\frac{17}{3}\)
12. 24
13. -24
14. \(\frac{7}{3}\)
15. -1