Section 2.2 Objectives

- Solve multi-step equations using algebra properties of equality.
- Solve equations that have no solution and equations that have infinitely many solutions.
- Solve equations with rational numbers.
- Solve application word problems by translating them into algebraic equations and solving them.
- Solve Geometry problems by writing and solving algebraic equations.
Introduction

In the previous section, you learned the basics of solving equations. However, the equations you solved required only one or two steps. In this section, you will learn to solve equations that require more than two steps. These equations are called multi-step equations.

An example of a multi-step equation is \(5(x + 3) = 6x - 2x + 1\). This example shows that a multi-step equation can contain parentheses. The example also shows that a multi-step equation can contain more than one variable term and can even contain variable terms on both sides of the equal sign.

To solve multi-step equations, you will continue to use the procedures that you learned in the last section: get the variable term alone, then get the variable itself alone. But there are a couple of additional steps that you must perform first. Those steps involve the distributive property and like terms. Let’s begin by reviewing these concepts.

### Distributive Property

Multiply the term outside the parentheses by each term inside the parentheses.

<table>
<thead>
<tr>
<th>Property</th>
</tr>
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<tbody>
<tr>
<td>(a(b + c))</td>
</tr>
<tr>
<td>(= ab + ac)</td>
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<table>
<thead>
<tr>
<th>Example</th>
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<tr>
<td>(2(3x + 8))</td>
</tr>
<tr>
<td>(= 6x + 16)</td>
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### Like Terms

Like terms have the same variable raised to the same exponent.

- **Combining Like Terms**
  - Add or subtract the coefficients. \(2x + 3x = 5x\)
  - Keep the variable and exponent the same. \(5x - 3x = 2x\)
SOLVING MULTI–STEP EQUATIONS

To solve multi-step equations, you begin by concentrating on one side of the equation at a time. You simplify each side of the equation by using the distributive property and by combining like terms. Then, if the equation contains two variable terms, you must eliminate one of them (either one) using inverse operations. After that, the goal will be the same as in the last section. You will get the variable term alone on one side of the equation. And last, you will get the variable itself alone on that same side of the equation. The entire process is described in the box below. Try to remember the key phrases that are underlined to help you perform the steps in order.

<table>
<thead>
<tr>
<th>SOLVING MULTI-STEP EQUATIONS</th>
</tr>
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<tbody>
<tr>
<td><strong>STEPS:</strong></td>
</tr>
<tr>
<td>1. Clear parentheses.</td>
</tr>
<tr>
<td>HOW? Use the Distributive Property.</td>
</tr>
<tr>
<td>2. Simplify each side of the equation.</td>
</tr>
<tr>
<td>HOW? Combine like terms on each side.</td>
</tr>
<tr>
<td>3. Get the equation to contain just ONE variable term.</td>
</tr>
<tr>
<td>HOW? If there are two variable terms, use inverse operations (+ or −) to eliminate either one.</td>
</tr>
<tr>
<td>4. Get the variable term ALONE on one side of the equation.</td>
</tr>
<tr>
<td>HOW? Use inverse operations (+ or −) to eliminate the constant from the side with the variable term.</td>
</tr>
<tr>
<td>5. Get the variable alone on one side of the equation.</td>
</tr>
<tr>
<td>HOW? Use inverse operations (× or ÷) to eliminate the coefficient in front of the variable.</td>
</tr>
<tr>
<td>6. Check the answer.</td>
</tr>
<tr>
<td>HOW? In the original equation, replace the variable(s) with the answer to see if it produces a true statement when simplified.</td>
</tr>
</tbody>
</table>

IMPORTANT: Perform the same operation on both sides of the equation.
**EXAMPLES:** Solve each equation.

1. \(3(4x + 2) = 30\)
   - **Parentheses:** Use the Distributive Property to clear the parentheses.
   - **Like Terms:** There are no like terms to combine on either side of the equation.
   - **ONE Variable Term:** There is just ONE variable term in the equation: \(12x\)
   - **Variable Term ALONE:** To get \(12x\) alone on the left side of the equal sign, subtract 6 from both sides of the equation.
   - \(12x = 24\)
   - \(\frac{12x}{12} = \frac{24}{12}\)
   - \(x = 2\)
   - **Check:** In the original equation, replace \(x\) with 2.
     - \(3(4x + 2) = 30\)
     - \(3(4 \cdot 2 + 2) = 30\)
     - \(3(8 + 2) = 30\)
     - \(3(10) = 30\)
     - \(30 = 30\) ✓
     - Since the check works, the solution is \(x = 2\).

2. \(4 + 5x - 3x = 5 - 9\)
   - **Parentheses:** There are no parentheses.
   - **Like Terms:** On the left side of the equal sign, combine like terms \(5x - 3x\).
     - On the right side of the equal sign, combine like terms \(5 - 9\).
   - **ONE Variable Term:** There is just ONE variable term in the equation: \(2x\)
   - **Variable Term ALONE:** To get \(2x\) alone on the left side of the equal sign, subtract 4 from both sides of the equation.
   - \(2x = -8\)
   - \(\frac{2x}{2} = \frac{-8}{2}\)
   - \(x = -4\)
   - **Check:** In the original equation, replace each \(x\) with \(-4\).
     - \(4 + 5x - 3x = 5 - 9\)
     - \(4 + 5(-4) - 3(-4) = 5 - 9\)
     - \(4 + (-20) + 12 = -4\)
     - \(-16 + 12 = -4\)
     - \(-4 = -4\) ✓
     - Since the check works, the solution is \(x = -4\).
3. \( x + 10 = -5x - 8 \)

Parentheses: There are no parentheses.

Like Terms: There are no like terms to combine on either side of the equation.

ONE Variable Term: The equation has two variable terms: \( x \) and \(-5x\).

We will use inverse operations to remove one of those terms. We decide to remove \(-5x\), so we perform the inverse and \(+5x\) to both sides of the equation.

\[
\begin{align*}
6x + 10 &= -8 \\
6x &= -18 \\
x &= -3
\end{align*}
\]

Check: In the original equation, replace each \( x \) with \(-3\).

Since the check works, the solution is \( x = -3 \).

4. \( 8x - 8 = 2(7x - 25) \)

Parentheses: Use the Distributive Property to clear the parentheses.

Like Terms: There are no like terms to combine on either side of the equation.

ONE Variable Term: The equation has two variable terms: \( 8x \) and \( 14x \).

We will use inverse operations to remove one of those terms. We decide to remove \( 8x \), so we perform the inverse and \(-8x\) from both sides of the equation.

\[
\begin{align*}
8x - 8 &= 14x - 50 \\
-8x &= 6x - 50 \\
-8 &= 6x \\
6x &= 50 \\
42 &= 6x \\
7 &= x
\end{align*}
\]

Check: In the original equation, replace each \( x \) with \( 7 \).

Since the check works, the solution is \( x = 7 \).
5. \(-4(1 + x) - 3x = -7 + 10\)

**Parentheses:** Use the Distributive Property to clear the parentheses.

\[-4 - 4x - 3x = -7 + 10\]

**Like Terms:** On the left side of the equal sign, combine like terms \(-4x - 3x\).

On the right side of the equal sign, combine like terms \(-7 + 10\).

\[-4 - 7x = 3\]

**ONE Variable Term:** There is just ONE variable term in the equation: \(-7x\).

Variable Term ALONE: To get \(-7x\) alone on the left side of the equal sign, add 4 to both sides of the equation.

\[\begin{align*}
-7x &= 3 \\
+4 &= +4 \\
\hline
-7x &= 7
\end{align*}\]

Variable Alone: To get \(x\) alone on the left side of the equal sign, divide by \(-7\) on both sides of the equation.

\[\begin{align*}
\frac{-7x}{-7} &= \frac{7}{-7} \\
\hline
x &= -1
\end{align*}\]

Check: In the original equation, replace each \(x\) with \(-1\). This check is left for you to complete.

6. \(-9 + 2 + 3x = 13 + 9x - 5x\)

**Parentheses:** There are no parentheses.

\[-9 + 2 + 3x = 13 + 9x - 5x\]

**Like Terms:** On the left side of the equal sign, combine like terms \(-9 + 2\).

On the right side of the equal sign, combine like terms \(9x - 5x\).

\[-7 + 3x = 13 + 4x\]

**ONE Variable Term:** The equation has two variable terms: \(3x\) and \(4x\). We will use inverse operations to remove one of those terms. We decide to remove \(4x\), so we perform the inverse \(-4x\) from both sides of the equation.

After simplifying, there is just ONE variable term: \(-1x\)

Variable Term ALONE: To get \(-1x\) alone on the left side of the equal sign, add 7 to both sides of the equation.

\[\begin{align*}
-7 + 3x &= 13 \\
+7 &= +7 \\
\hline
3x &= 20
\end{align*}\]

Variable Alone: To get \(x\) alone on the left side of the equal sign, divide by \(-1\) on both sides of the equation.

\[\begin{align*}
\frac{3x}{-1} &= \frac{20}{-1} \\
\hline
x &= -20
\end{align*}\]

Check: In the original equation, replace each \(x\) with \(-20\). This check is left for you to complete.

7. \(3(x - 2) - 5 = 7x + 3\)

**Parentheses:** Use the Distributive Property to clear the parentheses.

\[3x - 6 - 5 = 7x + 3\]

**Like Terms:** On the left side of the equal sign, combine like terms \(-6 - 5\).

**ONE Variable Term:** The equation has two variable terms: \(3x\) and \(7x\). We will use inverse operations to remove one of those terms. We decide to remove \(3x\), so we perform the inverse \(-3x\) from both sides of the equation.

After simplifying, there is just ONE variable term: \(4x\)

Variable Term ALONE: To get \(4x\) alone on the right side of the equal sign, subtract 3 from both sides of the equation.

\[\begin{align*}
3x - 11 &= 7x + 3 \\
-3x &= -3x \\
\hline
-11 &= 4x + 3 \\
-3 &= -3 \\
\hline
-14 &= 4x
\end{align*}\]

Variable Alone: To get \(x\) alone on the right side of the equal sign, divide by \(4\) on both sides of the equation. Simplify the fraction on the left side.

\[\begin{align*}
\frac{-14}{4} &= \frac{4x}{4} \\
\hline
-\frac{7}{2} &= x
\end{align*}\]

Check: In the original equation, replace each \(x\) with \(-\frac{7}{2}\).

This check is left for you to complete.
8. \(-5y + 30 = 5(3y - 2) - 12y\)  

**Parentheses:** Use the Distributive Property to clear the parentheses.  
**Like Terms:** On the right side of the equal sign, combine like terms \(15y - 12y\).  
**ONE Variable Term:** The equation has two variable terms: \(-5y\) and \(3y\).  
We will use inverse operations to remove one of those terms. We decide to remove \(-5y\), so we perform the inverse and \(+5y\) to both sides of the equation.  
After simplifying, there is just ONE variable term: \(8y\)  
**Variable Term ALONE:** To get \(8y\) alone on the right side of the equal sign, add 10 to both sides of the equation.  
**Variable Alone:** To get \(y\) alone on the right side of the equal sign, divide by 8 on both sides of the equation.  
**Check:** In the original equation, replace each \(y\) with 5.  
This check is left for you to complete.  
\[
\begin{align*}
-5y + 30 &= 5(3y - 2) - 12y \\
-5y + 30 &= 15y - 10 - 12y \\
-5y + 30 &= 3y - 10 \\
+5y &+ 5y \\
30 &= 8y - 10 \\
30 &= 8y - 10 + 10 \\
40 &= 8y \\
\frac{40}{8} &= \frac{8y}{8} \\
5 &= y
\end{align*}
\]
**Answer:** \(\emptyset\)  
We use the symbol \(\emptyset\) when there is No Solution.

9. \(7x - (4x - 2) = 3x + 11\)  

**Parentheses:** Use the Distributive Property to clear the parentheses.  
It may help to insert a 1 in front of the parentheses in order to distribute \(-1\).  
**Like Terms:** On the left side of the equal sign, combine like terms \(7x - 4x\).  
**ONE Variable Term:** The equation has two variable terms: \(3x\) on the left side of the equation and another \(3x\) on the right side of the equation.  
To remove \(3x\), we perform the inverse and \(-3x\) from both sides.  
Simplify the left side of the equation: \(3x - 3x\) is 0. Just the 2 remains.  
Simplify the right side of the equation: \(3x - 3x\) is 0. Just the 11 remains.  
The resulting equation has no variable, and has different numbers on each side.  
This is a false statement. It means that the original equation has No Solution.  
**Answer:** \(\emptyset\)  
We use the symbol \(\emptyset\) when there is No Solution.

10. \(4(3x - 8) = 7x + 5x - 32\)  

**Parentheses:** Use the Distributive Property to clear the parentheses.  
**Like Terms:** On the right side of the equal sign, combine like terms \(7x + 5x\).  
**ONE Variable Term:** The equation has two variable terms: \(12x\) on the left side of the equation and another \(12x\) on the right side of the equation.  
To remove \(12x\), we perform the inverse and \(-12x\) from both sides.  
Simplify each side. Just \(-32\) remains on each side.  
The resulting equation has no variable, and the same number is on both sides. This is a true statement. It means that every real number is a solution of the original equation.  
**Solution:** All Real Numbers


## SPECIAL CASES

<table>
<thead>
<tr>
<th>No Solution</th>
<th>All Real Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>When solving an equation:</td>
<td>When solving an equation:</td>
</tr>
<tr>
<td>▪ if the simplified equation is a <strong>False</strong> statement (like $5 = 9$),</td>
<td>▪ if the simplified equation is a <strong>True</strong> statement (like $5 = 5$),</td>
</tr>
<tr>
<td>▪ then the original equation has <strong>No Solution</strong> (represented by the symbol $\emptyset$).</td>
<td>▪ then the original equation has <strong>All Real Numbers</strong> as the solution.</td>
</tr>
</tbody>
</table>

**Note:** This means that there is no number that can replace the variable in the original equation to make it true.

**Note:** This means that every real number can replace the variable in the original equation to make it true.

---

### PRACTICE:
Solve each equation and check the solution. Be sure to write out all the algebra steps. If there is no solution, use the $\emptyset$ symbol.

1. $8 + 4 = x - 4x$
2. $5x = 2x - 27$
3. $7(x + 5) = -21$
4. $2a - 1 + 4 = 5$
5. $7x - 11 = 2x + 5$
6. $-6 = -3(x - 1)$
7. $-2x - 7 = 6x + 9$
8. $2x - 1.5x - 13 = 2$
9. $4(9 - 2x) = 4x$
10. $8x - 16 + 4 = 5x - 9x$
11. $2(y - 6) + 3 = -15$
12. $5x + 2x + 3 = -6 + 9 + 7x$
13. $2(3 - x) + 4x = -3 + 7$
14. $3x + 2(3x + 1) = 9x - 8$
15. $5 - (3x - 6) = -2x + 1$
16. $2(2x - 1) - 3 = -4(x + 4)$

### Answers:

1. $x = -4$
2. $x = -9$
3. $x = -8$
4. $a = 1$
5. $x = \frac{16}{5}$
6. $x = 3$
7. $x = -2$
8. $x = 30$
9. $x = 3$
10. $x = 1$
11. $y = -3$
12. All Real Numbers
13. $x = -1$
14. $\emptyset$
15. $x = 10$
16. $x = -11\frac{11}{8}$
SOLVING EQUATIONS WITH RATIONAL NUMBERS

You will continue to solve equations in this section. The only difference will be that the equations will contain fractions. We admit that working with fractions can be more time consuming and more tedious than working with integers. So for that reason, if we have an equation with fractions, our first step will be to eliminate the fractions. We will accomplish this by multiplying every term of the equation by the Least Common Denominator (LCD). Once the fractions are eliminated, we will solve the equation in the usual way.

**SOLVING EQUATIONS WITH RATIONAL NUMBERS**

If an equation contains one or more fractions, eliminate the fractions first.

Do this by multiplying every term on both sides of the equation by the Least Common Denominator (LCD).

**EXAMPLES:** Solve each equation.

1. Solve \(-\frac{2}{7}x = 4\)

   \[-\frac{2}{7}x = 4\] \quad \text{Determine the LCD:}
   \[\text{There is only one fraction, and its denominator is 7. So, the LCD is 7.}\]

   \[7 \left( -\frac{2}{7}x \right) = 7(4)\] \quad \text{Multiply both sides of the equation by 7, the LCD.}

   \[\frac{1}{1} \left( -\frac{2}{7}x \right) = 7(4)\] \quad \text{On the left side of the equation, write 7 as a fraction with a denominator of 1, then divide out common factors.}
   \[\text{Perform the multiplication on each side of the equation.}\]

   \[-2x = 28\] \quad \text{Notice that multiplying by the LCD eliminated the fractions.}

   \[\frac{2}{2} \cdot \frac{28}{-2}\] \quad \text{To get the variable alone, divide by \(-2\) on both sides of the equation.}

   \[x = -14\] \quad \text{This is the solution.}

   The check is left for you to complete.
2. Solve \( \frac{3}{4}x + x = 14 \)

\[
\frac{3}{4}x + x = 14
\]

Determine the LCD:
The there is only one fraction, and its denominator is 4. So, the LCD is 4.

Multiply every term on both sides of the equation by 4, the LCD.

\[
4 \left( \frac{3}{4}x + x \right) = 4 \cdot 14
\]

On the left side of the equation, write 4 as a fraction with a denominator of 1, then divide out common factors.

Perform the multiplications on each side of the equation.

Notice that multiplying by the LCD eliminated the fractions.

Combine like terms.

To get the variable alone, divide by 7 on both sides of the equation.

This is the answer.

Check:

To check the answer, replace each \( x \) in the original equation with 8.

\[
\frac{3}{4}(8) + 8 = 14
\]

Since the check works, the solution is \( x = 8 \).

3. Solve \( \frac{5}{8}x = -\frac{1}{3} \)

\[
\frac{5}{8}x = -\frac{1}{3}
\]

Determine the LCD:
The denominators of the fractions are 8 and 3. The LCD of 8 and 3 is 24.

Multiply every term on both sides of the equation by 24, the LCD.

\[
24 \left( \frac{5}{8}x \right) = 24 \left( -\frac{1}{3} \right)
\]

On both sides of the equation, write 24 as a fraction with a denominator of 1, then divide out common factors.

Perform the multiplications on each side of the equation.

Notice that multiplying by the LCD eliminated the fractions.

To get the variable alone, divide by 15 on both sides of the equation.

This is the solution. We leave the check to you.
4. Solve \(2x - \frac{8}{5} = \frac{16}{15}\)

\[
2x - \frac{8}{5} = \frac{16}{15} \\
15(2x) - 15 \left( \frac{8}{5} \right) = 15 \left( \frac{16}{15} \right) \\
15(2x) - 3 \left( \frac{8}{5} \right) = 1 \left( \frac{16}{15} \right) \\
30x - 24 = 16 \\
30x = 40 \\
x = \frac{40}{30} = \frac{4}{3}
\]

Determine the LCD: The denominators of the fractions are 5 and 15. The LCD of 5 and 15 is 15.

Multiply every term on both sides of the equation by 15, the LCD.

Write integers as fractions with a denominator of 1, then divide out common factors.

Perform the multiplications on each side.

Multiplying by the LCD eliminated the fractions.

To get the variable term 30x alone, add 24 to both sides of the equation.

To get \(x\) alone, divide by 30 on both sides of the equation. Then simplify the fraction.

This is the solution. We leave the check to you.

5. Solve \(\frac{3}{4}x - \frac{1}{5} = \frac{1}{2}x + \frac{3}{4}\)

\[
\frac{3}{4}x - \frac{1}{5} = \frac{1}{2}x + \frac{3}{4} \\
20 \left( \frac{3}{4}x \right) - 20 \left( \frac{1}{5} \right) = 20 \left( \frac{1}{2}x \right) + 20 \left( \frac{3}{4} \right) \\
5 \left( \frac{3}{4}x \right) - 4 \left( \frac{1}{5} \right) = 10 \left( \frac{1}{2}x \right) + 5 \left( \frac{3}{4} \right) \\
15x - 4 = 10x + 15 \\
-10x \\
5x = 15 \\
5x + 4 = 19 \\
5x = 19 \\
\frac{5x}{5} = \frac{19}{5} \\
x = \frac{19}{5}
\]

Determine the LCD: The denominators of the fractions are 2, 4, and 5. The LCD is 20.

Multiply every term on both sides of the equation by 20, the LCD.

Write integers as fractions with a denominator of 1, then divide out common factors.

Perform the multiplications on each side of the equation to eliminate the fractions.

There are two variable terms: 15x and 10x.

To get just one variable term, subtract 10x from both sides.

After simplifying, the equation contains just one variable term: 5x

To get the variable term 5x alone, add 4 to both sides of the equation.

To get \(x\) alone, divide by 5 on both sides.

This is the solution. We leave the check to you.
PRACTICE: Solve each equation and check the solution. Be sure to write out all the algebra steps. If there is no solution, use the \( \varnothing \) symbol.

1. \( \frac{3}{4}x = 12 \)
2. \( \frac{8}{9} = x - 1 \)
3. \( \frac{5}{11}x = -\frac{10}{11} \)
4. \( x - \frac{2}{3} = \frac{4}{3} \)
5. \( -\frac{4}{7}x = \frac{2}{9} \)
6. \( x - \frac{8}{9} = \frac{1}{3} \)
7. \( \frac{1}{2}x - \frac{2}{3} = 8 \)
8. \( \frac{4}{7}x - \frac{1}{2} = x \)
9. \( \frac{3}{5} + x = \frac{2}{3}x \)
10. \( \frac{1}{2}x + 10 = \frac{3}{4}x + 9 \)
11. \( \frac{3}{5}x - \frac{1}{6} = \frac{1}{2}x \)
12. \( \frac{7}{8}x - \frac{1}{3} = 2x - \frac{5}{6} \)
13. \( \frac{3}{4}x - \frac{1}{5} = \frac{2}{5}x + \frac{3}{4} \)
14. \( \frac{4}{3}x - \frac{1}{2} = \frac{1}{4}x + 3 \)

Answers:

1. \( x = 16 \)
2. \( x = \frac{1}{9} \)
3. \( x = -2 \)
4. \( x = 2 \)
5. \( x = -\frac{7}{18} \)
6. \( x = \frac{11}{9} \)
7. \( x = \frac{52}{3} \)
8. \( x = -\frac{7}{6} \)
9. \( x = -\frac{9}{5} \)
10. \( x = 4 \)
11. \( x = \frac{5}{3} \)
12. \( x = \frac{4}{9} \)
13. \( x = \frac{19}{7} \)
14. \( x = \frac{42}{13} \)
**Solving Application Problems**

Many situations in the real world can be modeled using mathematical equations. Solving an equation that represents a practical problem is usually straightforward, but translating the word problem into an algebraic equation takes more thought. Learning to write an equation to accurately represent some unique application requires practice. And although no single method will work for solving all applied problems, the following approach will help in the problem solving process.

### SOLVING WORD PROBLEMS

1. **Read the problem.**
   - Read until you understand it.
   - Highlight important information.
   - Draw a picture if helpful.

2. **Define a variable.**
   - Identify what you are asked to find.
   - Choose a variable to represent the unknown quantity.

3. **Write an algebraic equation.**
   - Break the sentences down and translate one part at a time into math.
   - Use key words to identify math operations.
   - Look for relationships among quantities.

4. **Solve the equation.**
   - Perform the algebraic steps to isolate the variable.

5. **Check the answer.**
   - Substitute the answer in the original equation.
   - Be sure the answer makes sense in the context of the problem.

6. **State the answer.**
   - Write a phrase or sentence giving the answer.

The examples and practice problems that follow deal with applications involving finance, business, geometry, education, health care, sports, nutrition, and auto mechanics. These problems provide just a sample of the many fields that have practical applications of algebra.
EXAMPLES: Write an algebraic equation for each word problem, then solve the problem to answer the question.

1. Katrina’s uncle loaned her $1200 to buy a computer. Katrina plans to pay her uncle $75 per month until the loan is paid off. How many months will it take Katrina to pay back the money?

Variable: \( m = \) the number of months it will take Katrina to pay back the money

Equation: \[
\text{monthly payment} \times \text{number of months} = \text{loan amount}
\]
\[
$75 \times m = $1200
\]

Solve: \[
75m = 1200
\]
\[
\frac{75m}{75} = \frac{1200}{75}
\]
\[
m = 16
\]

Check: \[
75m = 1200
\]
\[
75(16) = 1200 \checkmark
\]

Answer: It will take Katrina 16 months to pay back the loan.

2. A medical center is planning to hire a total of 54 nurses and CNAs (Certified Nursing Assistants). If the center needs twice as many CNAs as nurses, how many of each should they hire?

Variable: \( n = \) the number of nurses to hire

Equation: \[
\text{# of nurses} + \text{# of CNAs} = \text{total # nurses and CNAs}
\]
\[
n + 2n = 54
\]

Solve: \[
n + 2n = 54
\]
\[
3n = 54
\]
\[
\frac{3n}{3} = \frac{54}{3}
\]
\[
n = 18
\]

Check: \[
n + 2n = 54
\]
\[
18 + 2(18) = 54 \checkmark
\]

Answer: # of nurses = \( n = 18 \)
# of CNAs = \( 2n = 2(18) = 36 \)

The center should hire 18 nurses and 36 CNAs.
3. A company is ordering 350 t-shirts to sell at a fundraiser. The company wants to order the same number of large size t-shirts as medium size t-shirts, but only half as many small t-shirts. How many of each size should they order?

Variable: \( m \) = the number of medium size t-shirts

Equation: \[
\left( \frac{1}{2} m \right) + m + m = 350
\]

Solve: \[
\frac{1}{2} m + m + m = 350 \quad \text{The LCD is 2.}
\]

\[
2 \left( \frac{1}{2} m \right) + 2(m) + 2(m) = 2(350)
\]

Combine like terms.

\[
5m = 700
\]

Divide by 5 on both sides to get \( m \) alone.

\[
m = 140
\]

Check: We leave this check for you to complete.

Answer: # of large shirts = \( m = 140 \)

# of medium shirts = \( m = 140 \)

# of small shirts = \( \frac{1}{2} m = \frac{1}{2}(140) = 70 \)

The company should order 70 small shirts, 140 medium shirts, and 140 large shirts.

4. The perimeter of the triangle shown to the right is 26 inches. Determine the length of each side of the triangle.

Variable: \( x \) = the length of one side of the triangle

Equation: Recall that perimeter is the distance around the triangle.

\[
\frac{1}{2} x + 3x + 2x + 8 = 26
\]

Solve: \[
x + 3x + 2x + 8 = 26 \quad \text{Combine like terms on the left side of the equation.}
\]

\[
6x + 8 = 26
\]

To get the variable term 6\( x \) alone on the left side of the equal sign, subtract 8 from both sides of the equation.

\[
6x = 18
\]

\[
x = 3
\]

Check: We leave this check for you to complete.

Answer: 1\( \text{st} \) Side = \( x = 3 \)

2\( \text{nd} \) Side = \( 3x = 3(3) = 9 \)

3\( \text{rd} \) Side = \( 2x + 8 = 2(3) + 8 = 6 + 8 = 14 \)

The lengths of the sides of the triangle are 3 inches, 9 inches, and 14 inches.
5. The auto repair shop took 2.5 hours to repair Victoria’s car. The cost of the parts needed was $93, and the total bill was $248. What is the repair shop’s charge per hour for labor?

**Variable:** \( x = \) hourly charge for labor

**Equation:** 
\[
\text{Cost for Parts} + \text{Cost for Labor} = \text{Total Cost}
\]
\[
93 + (2.5)(x) = 248
\]

**Solve:**
\[
93 + 2.5x = 248 \\
-93 \\
2.5x = 155 \\
\frac{2.5}{2.5} \times \frac{x}{2.5} = \frac{155}{2.5} \Rightarrow x = 62
\]

**Check:** We leave this check for you to complete.

**Answer:** The repair shop’s charge for labor is $62 per hour.

6. Your grades on your first three math tests were 81, 76, and 78. You have one more test to take. What grade do you need on that last test in order to get an 80 average?

**Variable:** \( x = \) the grade you need on the last test

**Equation:** 
\[
\text{Average} = \frac{\text{Sum of Test Grades}}{\text{# of Tests}}
\]
\[
80 = \frac{81 + 76 + 78 + x}{4}
\]

**Solve:**
\[
80 = \frac{81 + 76 + 78 + x}{4} \\
4(80) = 4 \left( \frac{81 + 76 + 78 + x}{4} \right) \\
4(80) = \frac{81 + 76 + 78 + x}{1} \\
320 = 81 + 76 + 78 + x \\
320 = 235 + x \\
320 - 235 = 235 + x - 235 \\
85 = x
\]

**Check:** We leave this check for you to complete.

**Answer:** You will need to score 85 on the last test.
7. The perimeter of a soccer field at a local park is 260 feet. The length of the field is 36 feet more than the width. Find the dimensions of the soccer field.

**Variable:** \( w = \) the width of the soccer field

**Equation:** Recall that perimeter is the distance around the rectangular field. The formula for perimeter of a rectangle is \( P = 2L + 2W \).

\[
\text{Perimeter} = 2 \text{ Length} + 2 \text{ Width} \\
260 = 2(w+36) + 2w
\]

**Solve:**

\[
260 = 2(w+36) + 2w \\
260 = 2w + 72 + 2w \\
260 = 4w + 72 \\
-72 = -72 \\
188 = 4w \\
\frac{188}{4} = \frac{4w}{4} \\
47 = w
\]

**Check:** We leave this check for you to complete.

**Answer:**

Width of soccer field \( w = 47 \) feet.

Length of soccer field \( w + 36 = 47 + 36 = 83 \) feet.

The dimensions of the soccer field are 47 feet by 83 feet.

8. You have $65 in your saving account, and your brother has $140 in his savings account. You are depositing $15 per week to your account, and your brother is depositing $10 per week to his account. In how many weeks will you and your brother have the same amount of money?

**Variable:** \( w = \) the number of weeks until you and your brother have the same amount of money

**Equation:**

\[
\begin{align*}
\text{YOUR \ MONEY} \quad &\quad \text{= \ \text{BROTHER’S \ MONEY}} \\
\text{Current Balance} + \text{New Deposits} &\quad \text{= \ \ Current Balance} + \text{New Deposits} \\
65 + (15) (w) &\quad = \ 140 + (10) (w)
\end{align*}
\]

**Solve:**

\[
\begin{align*}
65 + 15w &= 140 + 10w \\
-10w &\quad -10w \\
65 + 5w &= 140 \\
-65 &\quad -65 \\
5w &= 75 \\
\frac{5w}{5} &\quad = \ \frac{75}{5} \\
w &= 15
\end{align*}
\]

**Check:** We leave this check for you to complete.

**Answer:** You and your brother will have the same amount of money in 15 weeks.
**PRACTICE:** Write an algebraic equation for each word problem, then solve the problem to answer the question.

1. Kevin plans to buy a used car. The monthly payments will be $259. If Kevin earns $9.25 per hour, how many hours must Kevin work each month to afford his car payment?

2. Each lap around a track is $\frac{1}{4}$ mile. How many laps would you have to run for a 3-mile workout?

3. The bottle of juice says that it has three times as much orange juice as apple juice. If the bottle contains 64 fluid ounces, how many ounces of orange juice does it contain?

4. A new animal hospital is planning to purchase a total of 36 cages for dogs, cats, and rabbits. They want to purchase the same number of cages for dogs as for cats, but only one-fourth as many cages for rabbits. How many of each cage should they buy?

5. The perimeter of the rectangle shown to the right is 96 feet. Determine the length and width of the rectangle.

6. Jim is planning to take some classes at the community college. Tuition is $110 per credit, plus $60 in fees. Jim has saved $1050 to take classes. How many credits can Jim afford to take with the money he saved?

7. Sara has $75 to spend on clothes. She wants to buy a pair of jeans that cost $27 and spend the rest on t-shirts. Each t-shirt costs $8. How many t-shirts can Sara afford to buy?

8. Your scores on your first two games of bowling were 134 and 115. What score do you need on your third game to get an average score of 125?

9. The perimeter of a rectangle is 50 centimeters. If the width of the rectangle is 9 centimeters less than the length, find the dimensions of the rectangle.

10. A class of 50 students is divided into two groups. One group has 8 more students than the other. How many students are in each group?

11. You and your friend work in telemarketing. You earn $76 per week, plus $16 for each sale you make. Your friend earns $60 per week, plus $20 for each sale she makes. How many sales do you and your friend each have to make in a week for your weekly salaries to be equal?

12. On the triangle to the right, sides $\overline{AB}$ and $\overline{BC}$ are the same length. What is the length of side $\overline{AC}$?
Answers:

1. Equation: \(9.25h = 259\)
   Solution: 28 hours

2. Equation: \(\frac{1}{4}x = 3\)
   Solution: 12 laps

3. Equation: \(a + 3a = 64\)
   Solution: 48 fluid ounces

4. Equation: \(c + c + \frac{1}{4}c = 36\)
   Solution: 16 cages for dogs, 16 cages for cats, 4 cages for rabbits

5. Equation: \(2(4x-7) + 2(x) = 96\)
   Solution: Length = 37 feet, Width = 11 feet

6. Equation: \(110c + 60 = 1050\)
   Solution: 9 credits

7. Equation: \(27 + 8s = 75\)
   Solution: 6 shirts

8. Equation: \(\frac{134 + 115 + x}{3} = 125\)
   Solution: 126

9. Equation: \(50 = 2L + 2(L - 9)\)
   Solution: Length = 17 cm, Width = 8 cm

10. Equation: \(s + (s + 8) = 50\)
    Solution: Group A = 21 students, Group B = 29 students

11. Equation: \(76 + 16s = 60 + 20s\)
    Solution: 4 sales

12. Equation: \(4x + 9 = 7x - 12\)
    Solution: 7
SECTION 2.2 SUMMARY
Multi-Step Equations

<table>
<thead>
<tr>
<th>SOLVING MULTI-STEP EQUATIONS</th>
<th>Determine the value of the variable that makes the equation true.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Example: Solve (2(x + 3) - 1 = 15 - 7x + 4x) (\frac{5x}{3} = 15) (5x = 10) (\frac{5x}{5} = 10) (x = 2)</td>
</tr>
<tr>
<td></td>
<td>1. <strong>Clear parentheses</strong> by using the Distributive Property. (2(x + 3) - 1 = 15 - 7x + 4x) (2x + 6 - 1 = 15 - 7x + 4x) (2x + 5 = 15) (2x = 10) (x = 5)</td>
</tr>
<tr>
<td></td>
<td>2. <strong>Combine like terms</strong> on each side of the equation. (2x + 5 = 15) (2x = 10) (x = 5)</td>
</tr>
<tr>
<td></td>
<td>3. Get just <strong>ONE variable term</strong> by using inverse operations. (\frac{5x}{3} = 15) (\frac{5x}{5} = 10)</td>
</tr>
<tr>
<td></td>
<td>4. Get the <strong>variable term ALONE</strong> on one side of the equation by using inverse operations. (\frac{5x}{3} = 15) (\frac{5x}{5} = 10)</td>
</tr>
<tr>
<td></td>
<td>5. Get the <strong>variable ALONE</strong> on one side of the equation by using inverse operations. (\frac{5x}{3} = 15) (\frac{5x}{5} = 10)</td>
</tr>
<tr>
<td></td>
<td><strong>NOTE</strong> If the result of this step is a: (\bullet) False statement (like (5 = 3)), then there is <strong>No Solution</strong> - (\emptyset). (\bullet) True statement (like (1 = 1)), then the solution is <strong>All Real Numbers</strong>. (\bullet) Value for Variable (like (x = 2)), then check your answer by substituting the value in the original equation in place of each variable. Simplify.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SOLVING EQUATIONS WITH RATIONAL NUMBERS</th>
<th>If an equation contains any fractions, eliminate them first.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Example: Solve (5 + \frac{2}{3}x = \frac{1}{2}x) (6(5) + 6 \left( \frac{2}{3}x \right) = 6 \left( \frac{1}{2}x \right))</td>
</tr>
<tr>
<td></td>
<td>1. Use the denominators of the fractions to determine the LCD. (6(5) + 6 \left( \frac{2}{3}x \right) = 6 \left( \frac{1}{2}x \right))</td>
</tr>
<tr>
<td></td>
<td>2. Multiply every term on both sides of the equation by the LCD. (30 + 4x = 3x) (continue solving as usual)</td>
</tr>
<tr>
<td></td>
<td><strong>NOTE</strong>: If this is done correctly, the fractions should be gone!</td>
</tr>
<tr>
<td></td>
<td>3. Continue solving using the procedure above.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>SOLVING APPLICATION PROBLEMS</th>
<th>Write the word problem as an algebraic equation, then solve the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Example: A moving truck charges a $75 fee plus $25 per day. If a person has a budget of $400, for how many days can he rent the moving truck? (d = ) the number of days the person can rent the truck</td>
</tr>
<tr>
<td></td>
<td>(\downarrow) Flat Fee + (\downarrow) Usage Cost = (\downarrow) Total Cost (75 + (25)(d) = 400)</td>
</tr>
<tr>
<td></td>
<td>(75 + 25d = 400) (-75) (-75) (25d = 325) (\frac{25d}{25} = \frac{325}{25}) (d = 13)</td>
</tr>
<tr>
<td></td>
<td>The person can rent the moving truck for 13 days.</td>
</tr>
</tbody>
</table>

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Solve each equation. If there is no solution, use the ∅ symbol.

1. \(2(x-1)=16\)
2. \(-3(n-1)=-27\)
3. \(4(6-2x)=-5x\)
4. \(5(3x+12)=-15\)
5. \(4x-9+10=25\)
6. \(3x-5x+11=17\)
7. \(5x+38=15+3\)
8. \(7+3x-8x=2-5\)
9. \(6x=x-20\)
10. \(3x+8=2x-7\)
11. \(10+5x=17-3x\)
12. \(3-4x+7=-7x+3x+10\)
13. \(2x-7x+5=7-12\)
14. \(2(a-4)+15=13\)
15. \(-4x+3(2x-5)=31\)
16. \(2(3-x)+4x=-3+7\)
17. \(4-3(x-4)=2x\)
18. \(5(x+3)=2x-6\)
19. \(2(6-4x)=25-9x\)
20. \(-7(x+2)+3x=-4x-5\)
21. \(8-(3x-3)=2x-4\)
22. \(3(a-1)=5a+3-2a\)
23. \(2(4-2x)=-2(x+5)\)
24. \(3(2x+1)+11=-2(2x-2)\)
Solve each equation. If there is no solution, use the ∅ symbol.

25. \( \frac{x}{6} = 8 \)  
30. \( \frac{1}{8} x - \frac{3}{4} = 6 \)

26. \( \frac{2}{9} x = -6 \)  
31. \( \frac{3}{4} - 2x = \frac{2}{3} x \)

27. \( -\frac{5}{7} = 3x + 1 \)  
32. \( \frac{3}{4} x - \frac{5}{6} = \frac{2}{3} x \)

28. \( x - \frac{3}{5} = \frac{4}{15} \)  
33. \( \frac{1}{6} x + \frac{3}{5} = 1 - \frac{3}{10} x \)

29. \( \frac{4}{5} = -\frac{4}{7} x \)  
34. \( \frac{2}{3} x - \frac{1}{4} = \frac{3}{8} x + \frac{1}{3} \)

Write an algebraic equation for each word problem, then solve the equation to answer the question.

35. Deon has taken a car loan for $1800. To pay the loan off in 12 months, how much would he have to pay per month?

36. CCBC is planning to purchase a total of 500 blue and gold balloons to decorate for graduation. They want three times as many blue balloons as gold balloons. How many of each should they buy?

37. Ken and Vicki drove 825 miles on their vacation. Ken drove twice as many miles as Vicki. How many miles did each drive?

38. The perimeter of the triangle shown to the right is 85 inches. Determine the length of each side of the triangle.

39. The admission fee at a summer carnival is $5.00. Each ride at the carnival costs $1.50. If James has $20 to spend, how many rides can he go on?

40. Carlton has a job offer for a position as a telemarketer. The job pays $60 per week, plus a $15 commission for each sale he makes. How many sales would Carlton have to make in a week in order to earn $240?

41. Kay’s grades on her first three projects were 92, 85, and 87. She has one more project to submit. What grade does she need on that last project in order to get a 90 average?

42. The cost of 2 tables and 6 chairs is $650. If the table costs $85 more than a chair, find the cost of each item.

43. A club is selling necklaces to raise money. It costs $65 to rent a booth to sell the necklaces. If the necklaces cost $5.50 to make each one, and you sell them for $8.75 each, how many necklaces must be sold for your income to equal your expenses?
### Answers to Section 2.2 Exercises

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<tr>
<th>Exercise</th>
<th>Equation</th>
<th>Solution</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>$x = 9$</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$n = 10$</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$x = 8$</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>$x = -5$</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>$x = 6$</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>$x = -3$</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>$x = -4$</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>$x = 2$</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>$x = -4$</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>$x = -15$</td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>$x = \frac{7}{8}$</td>
<td></td>
</tr>
<tr>
<td>12.</td>
<td>All Real Numbers</td>
<td></td>
</tr>
<tr>
<td>13.</td>
<td>$x = 2$</td>
<td></td>
</tr>
<tr>
<td>14.</td>
<td>$a = 3$</td>
<td></td>
</tr>
<tr>
<td>15.</td>
<td>$x = 23$</td>
<td></td>
</tr>
<tr>
<td>16.</td>
<td>$x = -1$</td>
<td></td>
</tr>
<tr>
<td>17.</td>
<td>$x = \frac{16}{5}$</td>
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<tr>
<td>18.</td>
<td>$x = -7$</td>
<td></td>
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<tr>
<td>19.</td>
<td>$x = 13$</td>
<td></td>
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<tr>
<td>20.</td>
<td>$\emptyset$</td>
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<tr>
<td>21.</td>
<td>$x = 3$</td>
<td></td>
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<tr>
<td>22.</td>
<td>$\emptyset$</td>
<td></td>
</tr>
<tr>
<td>23.</td>
<td>$x = 9$</td>
<td></td>
</tr>
<tr>
<td>24.</td>
<td>$x = -1$</td>
<td></td>
</tr>
<tr>
<td>25.</td>
<td>$x = 48$</td>
<td></td>
</tr>
<tr>
<td>26.</td>
<td>$x = -27$</td>
<td></td>
</tr>
<tr>
<td>27.</td>
<td>$x = -\frac{4}{7}$</td>
<td></td>
</tr>
<tr>
<td>28.</td>
<td>$x = \frac{13}{15}$</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>$x = -\frac{7}{5}$</td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>$x = 54$</td>
<td></td>
</tr>
<tr>
<td>31.</td>
<td>$x = \frac{9}{32}$</td>
<td></td>
</tr>
<tr>
<td>32.</td>
<td>$x = 10$</td>
<td></td>
</tr>
<tr>
<td>33.</td>
<td>$x = \frac{6}{7}$</td>
<td></td>
</tr>
<tr>
<td>34.</td>
<td>$x = 2$</td>
<td></td>
</tr>
<tr>
<td>35.</td>
<td>Equation: $12m = 1800$</td>
<td>Solution: $150$</td>
</tr>
<tr>
<td>36.</td>
<td>Equation: $g + 3g = 500$</td>
<td>Solution: Gold = 125, Blue = 375</td>
</tr>
<tr>
<td>37.</td>
<td>Equation: $v + 2v = 825$</td>
<td>Solution: Vicki = 275 miles, Ken = 550 miles</td>
</tr>
<tr>
<td>38.</td>
<td>Equation: $2x + x + 4 + 3x - 9 = 85$</td>
<td>Solution: 19 inches, 30 inches, 36 inches</td>
</tr>
<tr>
<td>39.</td>
<td>Equation: $5.00 + 1.5r = 20.00$</td>
<td>Solution: 10 rides</td>
</tr>
<tr>
<td>40.</td>
<td>Equation: $60 + 15s = 240$</td>
<td>Solution: 12 sales</td>
</tr>
<tr>
<td>41.</td>
<td>Equation: $92 + 85 + 87 + \frac{x}{4} = 90$</td>
<td>Solution: 96</td>
</tr>
<tr>
<td>42.</td>
<td>Equation: $2(c + 85) + 6c = 650$</td>
<td>Solution: Chair = $60, Table = $145</td>
</tr>
<tr>
<td>43.</td>
<td>Equation: $65 + 5.50n = 8.75n$</td>
<td>Solution: 20 necklaces</td>
</tr>
</tbody>
</table>
### Mixed Review

Sections 1.1 – 2.2

1. Evaluate \( |4+(-14)| \div \sqrt{25} \times (5-2^3) \).

2. Evaluate \( 12x-5y \) if \( x=\frac{3}{4} \) and \( y=-\frac{1}{10} \).

3. Simplify \( -9a+2-8b+11a+7b-5 \).

4. Simplify \( \frac{2}{3}(15x-21)-4(3x-5) \).

5. Is \( x=-3 \) a solution of \( 9x+7=5(x-1) \)?

6. Solve \( x+\frac{4}{5}=\frac{5}{8} \).

7. Solve \( 10-1.5x=4 \).

8. Solve \( -1=\frac{3}{4}x+8 \)

9. Translate the words into an algebraic equation. Then solve the equation. 
   
   \textbf{The sum of three times a number and 7 is 20. Determine the number.}

10. Translate the words into an algebraic equation. Then solve the equation. 
   
   \textbf{Two less than the product of 4 and a number is 38. Determine the number.}

### Answers to Mixed Review

1. \(-6\) \hspace{2cm} 6. \( x=-\frac{7}{40} \)

2. \(\frac{19}{2}\) \hspace{2cm} 7. \( x=4 \)

3. \(2a-b-3\) \hspace{2cm} 8. \( x=-12 \)

4. \(-2x+6\) \hspace{2cm} 9. \( n=-9 \)

5. Yes, \(-20=-20\). \hspace{2cm} 10. \( n=10 \)