2.2 Geometry

Geometry is the study of shapes and their mathematical properties. In this section, we will learn to calculate the perimeter, area, and volume of a few basic geometric shapes.

Perimeter

We begin with perimeter. The root word in perimeter is meter which is a measure. The prefix in perimeter is peri which means around. If we put these meanings together, we have the meaning of the word perimeter – to measure around a geometric shape. So, perimeter is just the sum of the lengths of all the sides of a geometric shape.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The perimeter of a geometric shape is the distance around the shape.</td>
<td>Perimeter is calculated by adding the lengths of all the sides of the shape.</td>
</tr>
<tr>
<td></td>
<td>Perimeter = ( a + b + c + d )</td>
</tr>
</tbody>
</table>

Example 1: Compute the perimeter of the given triangle.

To calculate the perimeter, we add the lengths of the sides.

Perimeter = 3 m + 2 m + 4 m

= 9 m

Practice 1: Compute the perimeter of the given triangle. Answer: 14 meters

**Example 2:** Compute the perimeter of the given trapezoid.

To calculate the perimeter, we add the lengths of the sides.

Perimeter = 12 m + 9 m + 11 m + 14 m

= 46 m

**Practice 2:** Compute the perimeter of the given trapezoid.  
**Answer:** 40 cm

**Watch it:** [http://youtu.be/avOo8P2bSSQ](http://youtu.be/avOo8P2bSSQ)

**Example 3:** Compute the perimeter of the given square.

In squares, all sides have the same length.  
So each side of the square in this problem is 18 ft.

**Method 1:** To calculate the perimeter, add the lengths of the sides.

\[
\text{Perimeter} = 18 \text{ ft} + 18 \text{ ft} + 18 \text{ ft} + 18 \text{ ft} \\
= 72 \text{ ft}
\]

**Method 2:** Another way to compute the perimeter is to multiply 4 times the length of one side.

\[
\text{Perimeter} = 4 \times 18 \text{ ft} \\
= 72 \text{ ft}
\]

Notice that this method worked out exactly the same as the last one.

Let’s use Method 2 to give a formula for easily finding the perimeter of a square.

\[
\text{Perimeter} = 4 \times \underline{\text{One Side}} \\
\]

\[
P = 4 \times \frac{\text{One Side}}{S}
\]
FORMULA: PERIMETER OF A SQUARE

\[ \text{Perimeter} = 4 \times \text{Side} \]
\[ P = 4 \times S \]
\[ P = 4S \]

Practice 3: Compute the perimeter of the given square.  
\[ \text{Answer:} \quad 28 \text{ ft} \]


Example 4: Compute the perimeter of the given rectangle.

\[ \text{Perimeter} = 5 \text{ in} + 5 \text{ in} + 3 \text{ in} + 3 \text{ in} \]
\[ = 10 \text{ in} + 6\text{ in} \]
\[ = 16 \text{ in} \]

Method 1: To calculate the perimeter, add the lengths of the sides.

Method 2: Another way to calculate the perimeter is to double the length, double the width, and add the results.

\[ \text{Perimeter} = (2 \times 5 \text{ in}) + (2 \times 3 \text{ in}) \]
\[ = 10 \text{ in} + 6\text{ in} \]
\[ = 16 \text{ in} \]

Notice that this method worked out exactly the same as the last one.
Let’s use Method 2 to develop a formula for finding perimeter of a rectangle.

\[
\text{Perimeter} = (2 \times 5 \text{ in}) + (2 \times 3 \text{ in})
\]

\[
= (2 \times \text{Bottom}) + (2 \times \text{RightSide})
\]

\[
= (2 \times \text{Length}) + (2 \times \text{Width})
\]

\[
P = (2 \times L) + (2 \times W)
\]

**FORMULA: PERIMETER OF A RECTANGLE**

**Practice 4:** Compute the perimeter of the given rectangle. **Answer:** 26 in

**Example 5:** Compute the perimeter of the given rectangle.

\[
P = (2 \times L) + (2 \times W)
\]

\[
P = (2 \times 4 \text{ cm}) + (2 \times 7 \text{ cm})
\]

\[
P = 8 \text{ cm} + 14 \text{ cm}
\]

\[
P = 22 \text{ cm}
\]
Practice 5:  Compute the perimeter of the given rectangle.  \[ \text{Answer: } 28 \text{ cm} \]

![Rectangle diagram]


Example 6:  Find the length of the missing side of the triangle whose perimeter is 28 ft.

![Triangle diagram]

We know the perimeter is 28 ft.  This means that the 3 sides added together equal 28 ft.

To find the length of the missing side, we will add the 2 sides that we know and subtract the sum from the perimeter.

\[
8 \text{ ft} + 14 \text{ ft} = 22 \text{ ft} \quad \text{Add the two sides that are given.} \\
\text{The sum is 22 ft.}
\]

\[
28 \text{ ft} - 22 \text{ ft} = 6 \text{ ft} \quad \text{Subtract the sum from the perimeter.} \\
\text{The missing side is 6 ft.}
\]

Practice 6:  Find the length of the missing side of the triangle whose perimeter is 40 ft.

![Triangle diagram]

Answer:  15 ft

Watch it:  [http://youtu.be/ezGwsjSpL08](http://youtu.be/ezGwsjSpL08)
Area

We have learned that perimeter measures the distance around a shape. Area, on the other hand, measures the surface inside the shape. Area gives us the amount of surface covered by a flat two-dimensional shape. The area, or surface of a shape, is measured in square units.

<table>
<thead>
<tr>
<th>Definition</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The area of a geometric shape is the amount of surface inside the shape.</td>
<td>Area is the number of square units covered by the shape.</td>
</tr>
</tbody>
</table>

Let’s look at area more closely and consider its meaning.

Suppose we want to know the area of the rectangle to the right.

We need to place squares with side length of one unit inside the rectangle. Try to think of placing square tiles on a floor. The rectangle to the right shows the result.

The area is the number of squares that are needed to cover the rectangle. The drawing to the right shows that the area of the rectangle is 15 square units.

But what if we had a very large rectangle, as large as the floor in this room? Drawing squares inside a rectangle that large and counting the squares would be a bit tedious, don’t you think? There must be an easier way to determine area. Let’s find it . . .
Let’s use the previous rectangle again.

Notice that there are 5 squares along the top row, and 3 squares along the right side. So, rather than counting the squares one by one, we can simply multiply $5 \times 3$ to get our answer, 15 square units.

Let’s turn this into a formula for finding the area of a rectangle.

\[
\text{Area} = \frac{5 \text{ units}}{} \times \frac{3 \text{ units}}{} \\
= \frac{\text{Length}}{} \times \frac{\text{Width}}{} \\
A = \frac{L}{\text{units}} \times \frac{W}{\text{units}}
\]

**Example 7:** Compute the area of the given rectangle.

![Rectangle diagram](image)

Let’s use the formula. Notice that the answer can be written in either of the two ways shown. The length and width were given in inches, but the answer must be given as square inches or inches$^2$.

\[A = 7 \text{ in} \times 6 \text{ in} \quad A = 42 \text{ square inches} \quad \text{OR} \quad 42 \text{ in}^2\]

You know why, right? Because when we say the area of the rectangle is 42 square inches, it means that 42 squares cover the surface. If you like, you can count them in the drawing to the right.
**Practice 7:** Compute the area of the given rectangle.  
Answer: 35 in²

![Rectangle Diagram](image)

**Watch it:** [http://youtu.be/opAyOsJT64](http://youtu.be/opAyOsJT64)

---

**Example 8:** Compute the area of the given square.

![Square Diagram](image)

Similar to a rectangle, to calculate the area of a square we multiply the length and the width. Remember that all sides have the same length on a square. So, the length and the width are both 14 ft.

\[
\text{Area} = \text{Length} \times \text{Width} \\
= 14\text{ft} \times 14\text{ft} \\
= 196\text{ft}^2
\]

---

**Practice 8:** Compute the area of the given square.  
Answer: 49 ft²

![Square Diagram](image)

**Watch it:** [http://youtu.be/pnptloscTPg](http://youtu.be/pnptloscTPg)
Example 9: Compute the area of the given triangle.

![Triangle Diagram]

In a triangle we call the length the base. In the drawing above, the base is 9 in. And we call the width the height. In the drawing above, the height is 4 in.

Notice that the height must be drawn straight up from the base. Think of a flag pole that has been placed in the ground.

To compute the area of a triangle, we will begin as we did with the rectangle. We will multiply the length and width, now called the base and height.

Then there will be just one additional step. We will divide the result by 2. Why divide by 2? Look at the drawing to the right. A triangle is half of a rectangle!

Let’s perform the computations now to get the area of the triangle.

\[
\text{Area} = (\text{Base} \times \text{Height}) \div 2 \\
= (9 \text{ in} \times 4 \text{ in}) \div 2 \\
= 36 \text{ in}^2 \div 2 \\
= 18 \text{ in}^2
\]

Practice 9: Compute the area of the given triangle.  

Answer: 90 in$^2$

Watch it: [http://youtu.be/TKm1toaHJDA](http://youtu.be/TKm1toaHJDA)
**FORMULA: AREA OF A TRIANGLE**

\[
\begin{align*}
\text{Area} &= (\text{Base} \times \text{Height}) \div 2 \\
A &= (B \times H) \div 2 \\
A &= (BH) \div 2
\end{align*}
\]

**Example 10:** Determine the area of the given triangle.

In this triangle, the base is 6 m and the height is 2 m.

As we said in the previous example, notice that the height is drawn straight up from the base. Think of the flag pole placed in the ground. Also notice that the height is not a side of the triangle. That’s okay – it is drawn straight up from the base to the highest point on the triangle. So, it is the height.

The 3 m side is not used in the calculation of the area.

To compute the area of the triangle, we will multiply the base and height. Then we will divide the result by 2.

Let’s perform the computations to get the area of the triangle.

\[
\begin{align*}
\text{Area} &= (\text{Base} \times \text{Height}) \div 2 \\
&= (6 \text{ m} \times 2 \text{ m}) \div 2 \\
&= 12 \text{ m}^2 \div 2 \\
&= 6 \text{ m}^2
\end{align*}
\]
**Practice 10:** Determine the area of the given triangle.  

![Triangle Diagram](image)

**Answer:** $150 \text{ m}^2$

**Watch it:** [http://youtu.be/5xxmgH36zUo](http://youtu.be/5xxmgH36zUo)

**Volume**

We have learned that *perimeter* measures the *distance around* a shape and that *area* measures the *surface inside* a shape. Now we are ready to study *volume*, the final topic in this section.

The major difference is that *volume* applies to *three*-dimensional objects, whereas *perimeter* and *area* apply to flat *two*-dimensional shapes. Do you see the difference in the chart below?

<table>
<thead>
<tr>
<th>Perimeter and Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Rectangle" /></td>
<td><img src="image" alt="Box" /></td>
</tr>
<tr>
<td><img src="image" alt="Triangle" /></td>
<td><img src="image" alt="Cuboid" /></td>
</tr>
<tr>
<td><img src="image" alt="Parallelogram" /></td>
<td><img src="image" alt="Triangular Prism" /></td>
</tr>
<tr>
<td><img src="image" alt="Trapezoid" /></td>
<td></td>
</tr>
</tbody>
</table>

Volume measures the amount of *space within* a three-dimensional geometric object. For example, a container of juice is a three-dimensional object. The volume of the container is the amount of juice the container can hold.

Volume is measured in **cubic units**.
## VOLUME

<table>
<thead>
<tr>
<th>Definition</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The volume of a</td>
<td>Volume is the number of</td>
</tr>
<tr>
<td>geometric object</td>
<td>cubic units</td>
</tr>
<tr>
<td>is the amount of</td>
<td>it takes to fill the object.</td>
</tr>
<tr>
<td>space within the object.</td>
<td>Volume = # of cubic units</td>
</tr>
</tbody>
</table>

Let’s look at volume more closely and consider its meaning.

Suppose we want to know the volume of the object to the right.

Imagine placing cubes inside the object. Try to think of putting dice inside a box. The drawing to the right shows how this would look.

The volume is the number of cubes (dice) that are needed to fill the object (box). The drawing to the right shows that there are 12 cubes facing you. There are 12 more cubes behind those.

So, the volume of the object is 24 cubic units or 24 units$^3$.

But drawing and counting cubes is not the way we want to find the volume of objects. That would be too tedious. Let’s examine an easier way using the same object as above . . .
Notice that there are 3 cubes along the bottom – this is the length. There are 2 cubes along the right side – this is the width. And the object is 4 cubes high – this is the height.

So, rather than counting the cubes one by one, we can multiply:

\[
\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}
\]
\[
= 3 \text{ units} \times 2 \text{ units} \times 4 \text{ units}
\]
\[
= 24 \text{ cubic units \ OR \ } 24 \text{ units}^3
\]

We get the same answer as above.

---

**Example 11:** Determine the volume of the given rectangular solid.

\[
\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}
\]
\[
= 9 \text{ units} \times 2 \text{ units} \times 4 \text{ units}
\]
\[
= 72 \text{ cubic units \ OR \ } 72 \text{ units}^3
\]
Practice 11: Determine the volume of the given rectangular solid.

Answer: 600 units$^3$

Watch it: [http://youtu.be/1ExjGSm3LPw](http://youtu.be/1ExjGSm3LPw)

Example 12: Determine the volume of the given cube.

In a cube the length, width, and height are equal.

\[
Volume = \text{Length} \times \text{Width} \times \text{Height}
\]

\[
= 9 \text{ units} \times 9 \text{ units} \times 9 \text{ units}
\]

\[
= 729 \text{ cubic units OR } 729 \text{ units}^3
\]

Practice 12: Determine the volume of the given cube.  

Answer: 2197 units$^3$

Watch it: [http://youtu.be/CD9CRXhm0Ew](http://youtu.be/CD9CRXhm0Ew)
In this section, you have studied how to find the perimeter, area, and volume of geometric shapes. Before you begin the exercises that follow, let’s review what you have learned.

<table>
<thead>
<tr>
<th>Description</th>
<th>Perimeter</th>
<th>Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Definition</strong></td>
<td>Distance around a 2-dimensional shape</td>
<td>Surface inside a 2-dimensional shape</td>
<td>Space within a 3-dimensional object</td>
</tr>
<tr>
<td><strong>Measurement</strong></td>
<td># of units around shape</td>
<td># of square units (or units²) covering shape</td>
<td># of cubic units (or units³) filling object</td>
</tr>
<tr>
<td>Note: add all sides</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Special Formulas</strong></td>
<td>Square: ( P = 4 \times S )</td>
<td>Rectangle: ( A = L \times W )</td>
<td>Rectangular Solid / Cube</td>
</tr>
<tr>
<td></td>
<td>( S )</td>
<td>( L )</td>
<td>( V = L \times W \times H )</td>
</tr>
<tr>
<td></td>
<td>( S )</td>
<td>( W )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( L )</td>
<td>( L )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( W )</td>
<td>( W )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( H )</td>
<td>( H )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( L )</td>
<td>( L )</td>
<td></td>
</tr>
<tr>
<td><strong>Watch All:</strong></td>
<td><a href="http://youtu.be/ZYiDezuoDOs">http://youtu.be/ZYiDezuoDOs</a></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.2 Geometry Exercises

1. Find the perimeter of a triangle if the lengths of the sides are 4 in, 5 in, and 6 in.

2. Find the perimeter of a square if the length of each side is 4 cm.

3. Find the perimeter of a rectangle if the length is 5 feet and the width is 2 feet.

4. Find the perimeter of a triangle if the lengths of the sides are 12 km, 13 km, and 20 km.

5. Find the area of a square if the length of each side is 3 yards.

6. Find the area of a rectangle if the length is 3 miles and the width is 10 miles.

7. Find the area of a triangle with base 14 inches and height 8 inches.

8. Find the area of a rectangle, given that the length of the shorter side is 2 feet and the length of the longer side is 8 feet.

9. Find the volume of a cube if the length of each side is 5 yards.

10. Find the volume of a rectangular solid if the length is 2 cm, the width is 3 cm, and the height is 12 cm.

11. Find the perimeter of the trapezoid below.

   ![Trapezoid](image)

12. Find the perimeter of the parallelogram below. In a parallelogram, the opposite sides have the same length.

   ![Parallelogram](image)

13. Find the perimeter of the hexagon below. Each side has a length of 7 yards.

   ![Hexagon](image)
14. Find the length of the missing side of the triangle whose perimeter is 83 ft.

15. Find the area of the square below.

16. Find the area of the triangle below.

17. Find the area of the triangle below.

18. Find the volume of the cube below.
19. Find the volume of the rectangular solid below.

![Rectangular Solid](image)

- Length: 4 m
- Width: 1 m
- Height: 3 m

Volume: $V = l \times w \times h = 4 \times 1 \times 3 = 12 \text{ m}^3$

20. Name the geometric shape and find its perimeter.

![Rectangle](image)

- Length: 7 ft
- Width: 3 ft

Perimeter: $P = 2(l + w) = 2(7 + 3) = 20 \text{ ft}$

21. Name the geometric shape and find its perimeter.

![Triangle](image)

- Base: 9 in
- Sides: 5 in and 6 in

Perimeter: $P = 9 + 5 + 6 = 20 \text{ in}$

22. Name the geometric shape and find its perimeter.

![Square](image)

- Side: 2 cm

Perimeter: $P = 4s = 4 \times 2 = 8 \text{ cm}$

23. Name the geometric shape and find its area.

![Rectangle](image)

- Length: 6 yd
- Width: 10 yd

Area: $A = l \times w = 6 \times 10 = 60 \text{ yd}^2$
2.2 Geometry Exercise Answers

1. 15 in
2. 16 cm
3. 14 ft
4. 45 km
5. 9 yds²
6. 30 mi²
7. 56 in²
8. 16 ft²
9. 125 yds³
10. 72 cm³
11. 34 m
12. 18 in
13. 42 yds
14. 22 ft
15. 121 cm²
16. 30 in²
17. 54 cm²
18. 1000 ft³
19. 12 m³
20. Rectangle; 20 ft
21. Triangle; 20 in
22. Square; 8 cm
23. Rectangle; 60 yds²
Translate the word phrase into a math expression.
1. $-15$ increased by $42$
2. the quotient of $24$ and $6$
3. $37$ subtracted from $10$
4. the product of $2$ cubed and $4$

Translate the word phrase into a math expression and find the value of the expression.
5. $6$ more than the product of $3$ and $-8$
6. $3$ less than twice $9$
7. the quotient of $-50$ and the sum of $6$ and $4$
8. $5$ times the difference of $12$ and $9$
9. the sum of $7$ squared and $8$

Solve each Geometry problem.
10. Determine the perimeter of a triangle if the lengths of the sides are $8$ feet, $10$ feet, and $16$ feet.
11. Determine the perimeter of a rectangle if the length is $9$ kilometers and the width is $11$ kilometers.
12. Determine the area of a square if the length of each side is $7$ inches.
13. Determine the area of a triangle whose base is $6$ centimeters and whose height is $25$ centimeters.
14. Determine the volume of a cube if the length of each side is $4$ meters.

15. The perimeter of the triangle below is $18$ feet. Determine the missing length.

16. Find the perimeter of the trapezoid below.

17. Name the shape below and find its perimeter.

18. Name the shape below and find its perimeter.

19. Name the shape below and find its area.

20. Find the area of the triangle below.

21. Find the area of the triangle below.

22. Find the volume of the rectangular solid.
### Mid-Chapter 2 Review Answers

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$-15 + 42$</td>
<td>12.</td>
</tr>
<tr>
<td>2.</td>
<td>$24 \div 6$</td>
<td>13.</td>
</tr>
<tr>
<td>3.</td>
<td>$10 - 37$</td>
<td>14.</td>
</tr>
<tr>
<td>4.</td>
<td>$2^3 \times 4$</td>
<td>15.</td>
</tr>
<tr>
<td>5.</td>
<td>$6 + (3 \times -8) = -18$</td>
<td>16.</td>
</tr>
<tr>
<td>6.</td>
<td>$2 \times 9 - 3 = 15$</td>
<td>17.</td>
</tr>
<tr>
<td>7.</td>
<td>$-50 \div (6 + 4) = -5$</td>
<td>18.</td>
</tr>
<tr>
<td>8.</td>
<td>$5 \times (12 - 9) = 15$</td>
<td>19.</td>
</tr>
<tr>
<td>9.</td>
<td>$7^2 + 8 = 57$</td>
<td>20.</td>
</tr>
<tr>
<td>10.</td>
<td>$34 \text{ feet}$</td>
<td>21.</td>
</tr>
<tr>
<td>11.</td>
<td>$40 \text{ kilometers}$</td>
<td>22.</td>
</tr>
</tbody>
</table>