CHAPTER 4: DECIMALS

CHAPTER 4 CONTENTS

4.1 Introduction to Decimals
4.2 Converting between Decimals and Fractions
4.3 Addition and Subtraction of Decimals
4.4 Multiplication and Division of Decimals
4.5 Metric Measurement
4.6 Applications
4.1 Introduction to Decimals

Our numbering system is based on powers of 10 and is therefore called the decimal number system, from the Latin root *decem*, meaning ten. In this chapter, we will study what decimal values represent and how they are connected to fractions. We will also learn how to round decimals and perform basic arithmetic operations on decimals. Finally, we will look at some applications of decimals to the metric system, geometry, finance, and statistics.

We use decimals every time we talk about money. When we get a bill at a restaurant or buy something at the store or online, we rarely must pay an amount that is a whole number dollar amount. You might buy a large iced coffee for $2.75 or download a Coldplay album for $13.99. In the previous chapter, we learned about fractions and mixed numbers that had whole number parts and fractional parts. The decimal 2.75 expresses a similar idea: you have two wholes (two whole dollars) and 75 cents, which is \( \frac{75}{100} \) of another dollar. In fact, \( 2.75 = 2 \frac{75}{100} \). So the numbers to the right of the decimal point represent a part of a whole in the same way that the fractional part of a mixed number does.

Of course, the use of decimals is not limited to money. We can calculate a baseball player’s batting average to be 0.286 or measure the weight of a bag of imported dehydrated lima beans to be 12.125 pounds.

When referring to decimals, we talk about **place values** just like we do with whole numbers. When we read the number 3,125 we say “three thousand, one hundred twenty-five”. That really means we have 3 thousands, plus 1 hundred, plus 2 tens, plus 5 ones. There are also place values for the numbers to the right of the decimal point as in the diagram below:

\[
\begin{array}{cccccc}
\text{thousands} & \text{hundreds} & \text{tens} & \text{ones} & \text{tenths} & \text{thousandths} \\
\end{array}
\]

Notice how the values behind (to the right of) the decimal point end in *ths*. Later, we will see why this is the case, but for now let’s say that it pertains to the connection between decimals and fractions. Also notice that there is no “*oneths*” place; the first place to the right of the decimal point is the **tenths** place. Although they are not listed in the diagram above, after the thousandths place come the ten thousandths, then the hundred thousandths, then the millionths, and so on.
Example 1: Consider the number 321.4758

<table>
<thead>
<tr>
<th>Place</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hundreds</td>
<td>3</td>
</tr>
<tr>
<td>Tens</td>
<td></td>
</tr>
<tr>
<td>Ones</td>
<td></td>
</tr>
<tr>
<td>Tenths</td>
<td>4</td>
</tr>
<tr>
<td>Hundredths</td>
<td></td>
</tr>
<tr>
<td>Thousandths</td>
<td>7</td>
</tr>
<tr>
<td>Ten Thousandths</td>
<td>5</td>
</tr>
</tbody>
</table>

a. What digit is in the *hundreds* place? **Answer:** The digit 3 is in the hundreds place.
b. What digit is in the *tenths* place? **Answer:** The digit 4 is in the tenths place.
c. What digit is in the *thousandths* place? **Answer:** The digit 5 is in the thousandths place.
d. Name the place value of the digit 7. **Answer:** The digit 7 is in the *hundredths* place.

Practice 1: Consider the number 9,056.1437

<table>
<thead>
<tr>
<th>Place</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ones</td>
<td>7</td>
</tr>
<tr>
<td>Tens</td>
<td>5</td>
</tr>
<tr>
<td>Tenths</td>
<td>6</td>
</tr>
<tr>
<td>Hundredths</td>
<td>0</td>
</tr>
<tr>
<td>Thousandths</td>
<td>1</td>
</tr>
<tr>
<td>Ten Thousandths</td>
<td>4</td>
</tr>
<tr>
<td>Hundredths</td>
<td>3</td>
</tr>
<tr>
<td>Thousandths</td>
<td>7</td>
</tr>
</tbody>
</table>

a. Which digit is in the *hundreds* place? **Answer:** 0
b. Which digit is in the *hundredths* place? **Answer:** 4
c. Name the place value of the digit 5. **Answer:** Tens
d. Name the place value of the digit 7. **Answer:** Ten thousandths

**Answers:** a. 0  b. 4  c. tens  d. ten thousandths


Example 2: Consider the number 3,145.097

<table>
<thead>
<tr>
<th>Place</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ones</td>
<td>3</td>
</tr>
<tr>
<td>Tens</td>
<td>1</td>
</tr>
<tr>
<td>Tenths</td>
<td>4</td>
</tr>
<tr>
<td>Hundredths</td>
<td>5</td>
</tr>
<tr>
<td>Thousandths</td>
<td>0</td>
</tr>
<tr>
<td>Ten Thousandths</td>
<td>9</td>
</tr>
<tr>
<td>Hundredths</td>
<td>7</td>
</tr>
</tbody>
</table>

a. What digit is in the *thousands* place? **Answer:** 3
b. What digit is in the *hundreds* place? **Answer:** 1
c. What digit is in the *tenths* place? **Answer:** 0
d. Name the place value of the digit 7. **Answer:** Thousandths

Practice 2: Consider the number 35,467.2901

<table>
<thead>
<tr>
<th>Place</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ones</td>
<td>7</td>
</tr>
<tr>
<td>Tens</td>
<td>6</td>
</tr>
<tr>
<td>Tenths</td>
<td>2</td>
</tr>
<tr>
<td>Hundredths</td>
<td>9</td>
</tr>
<tr>
<td>Thousandths</td>
<td>0</td>
</tr>
<tr>
<td>Ten Thousandths</td>
<td>1</td>
</tr>
</tbody>
</table>

a. Which digit is in the *ones* place? **Answer:** 7
b. Which digit is in the *tenths* place? **Answer:** 2
c. Name the place value of the digit 5. **Answer:** Thousands
d. Name the place value of the digit 0. **Answer:** Thousandths

**Answers:** a. 7  b. 2  c. thousands  d. thousandths

Watch It: [http://youtu.be/3drY204tHNs](http://youtu.be/3drY204tHNs)
Ordering Decimal Numbers

Sometimes it is necessary to compare decimal numbers. Note that adding 0 after the rightmost digit does not change the value of the decimal number. As examples, notice:

\[ 2.75 = 2.750 \quad \text{AND} \quad 0.6 = 0.60 = 0.600 = 0.6000 = \ldots \text{and so on.} \]

We will see why this is true when we investigate the relationship between decimal numbers and fractions.

<table>
<thead>
<tr>
<th>COMPARING DECIMALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Start from the left and compare digits in the corresponding place value positions.</td>
</tr>
<tr>
<td>2. Moving from left to right, the first number with a greater digit in the corresponding place position is the greater number.</td>
</tr>
</tbody>
</table>

**Example 3:** Compare the decimal numbers by filling in the blank with \(<\), \(>\), or \(=\).

\[ 0.6 \quad ____ \quad 0.59 \]

The first place from the left where their digits differ is in the tenths place. Compare the digits:

\[ 0.6 \quad ____ \quad 0.59 \]

Since 6 is greater than 5, 0.6 is greater than 0.59 and we write: \(0.6 > 0.59\).

**Practice 3:** Compare the decimal numbers by filling in the blank with \(<\), \(>\), or \(=\).

\[ 0.0792 \quad ____ \quad 0.079 \]

**Watch It:** [http://youtu.be/BrtI5lpf0o8](http://youtu.be/BrtI5lpf0o8)  
**Answer:** \(0.0792 > 0.079\)

**Example 4:** Compare the decimal numbers by filling in the blank with \(<\), \(>\), or \(=\).

\[ 3.9518 \quad ____ \quad 3.9523 \]

Notice these two decimal numbers have the same value in the ones place, the tenths place, and the hundredths place. The first place from the left where their digits differ is in the thousandths place. Compare the digits there:

\[ 3.9518 \quad ____ \quad 3.9523 \]

Since 1 is less than 2, 3.9518 is less than 3.9523 and we write: \(3.9518 < 3.9523\).

**Practice 4:** Compare the decimal numbers by filling in the blank with \(<\), \(>\), or \(=\).

\[ 10.601 \quad ____ \quad 10.61 \]

**Watch It:** [http://youtu.be/0w3Rc7KmB3M](http://youtu.be/0w3Rc7KmB3M)  
**Answer:** \(10.601 < 10.61\)
**Example 5:** Write the decimal numbers in order from least to greatest:

\[
31.025 \quad 31 \quad 31.205
\]

Notice we can write 31 as 31.000. Now compare digits in the three numbers “place-by-place” and rewrite from least to greatest as:

\[
31 \quad 31.025 \quad 31.205
\]

**Practice 5:** Write the decimal numbers in order from least to greatest:

\[
0.572 \quad 0.5719 \quad 0.5
\]

**Watch It:** [http://youtu.be/0MhQD-WSYtU](http://youtu.be/0MhQD-WSYtU)  
**Answer:** 0.5, 0.5719, 0.572

**Example 6:** The birth weights (in pounds) of quintuplet babies were:

\[
2.375 \quad 2.125 \quad 2.8125 \quad 2.8 \quad 2.25
\]

Find the median birth weight.

Start by listing the data values in order from least to greatest (by comparing their digits place-by-place):

\[
2.125 \quad 2.25 \quad 2.375 \quad 2.8 \quad 2.8125
\]

Because there are an odd number of data values (5), there is one middle value: 2.375.

**Answer:** The median weight is 2.375 pounds.

**Practice 6:** A grocery store sells bananas in bags with each bag containing approximately two pounds of bananas. Find the median weight for seven bags if their actual weights are:

\[
2.01 \quad 2.1 \quad 1.98 \quad 1.987 \quad 1.999 \quad 2.013 \quad 2.05
\]

**Watch It:** [http://youtu.be/8MZDHk727VQ](http://youtu.be/8MZDHk727VQ)  
**Answer:** 2.01 pounds

**Estimating Numbers**

Sometimes, we don’t need exact numbers; approximating or estimating is often good enough. If you are at the supermarket and notice that a box of Fructose Flakes cereal costs $4.79, you could estimate its cost to be about $5. Doing so would make it easier to calculate that four boxes will cost you roughly $20. Now, this is not an exact answer, but it’s a pretty close estimate. We don’t have to bother making a precise calculation if we just want to know approximately how much four boxes of this cereal would cost.

So how would we estimate a number like 2,132? The easiest answer would be to say that it is approximately 2,000 although you could also say that it is approximately 2,100 or even 2,130. But 2,000 is the “simplest” (for use in computations) estimate of 2,132. In this course, we will estimate numbers in the following way:
**HOW TO ESTIMATE**

1. Determine the left-most non-zero digit.

2. Look at the digit immediately to its right.
   - If that digit is less than 5, then the left-most digit remains unchanged.
   - If that digit is 5 or more, then we add 1 (round up) to the left-most digit.

3. Hold the remaining digits in place with appropriate zeros.

**Example 7:** Estimate 348

1. The left-most non-zero digit is the 3 in the hundreds place.
2. The digit immediately to its right is 4, which is less than 5; thus, the 3 remains unchanged.
3. Replace both of the digits to the right of the 3 with a 0 to keep the 3 in the hundreds place: 300

Answer: The estimate for 348 is 300.

**Practice 7:** Estimate 1,234  
**Answer:** 1,000

**Watch It:** [http://youtu.be/GGXjWqy1xh4](http://youtu.be/GGXjWqy1xh4)

**Example 8:** Estimate 4,897

1. The left-most non-zero digit is the 4 in the thousands place.
2. The digit immediately to its right is 8, which is more than 5; thus, we add 1 to (“bump up” by 1) the 4, making 5.
3. Replace each of the three digits to the right of the 5 with a 0 to keep the 5 in the thousands place: 5000

Answer: The estimate for 4,897 is 5000.

**Practice 8:** Estimate 37.5  
**Answer:** 40

**Watch It:** [http://youtu.be/gOeU9SjyDlw](http://youtu.be/gOeU9SjyDlw)
**Example 9:** Estimate 9,502

1. The left-most non-zero digit is the 9 in the thousands place.
2. The digit immediately to its right is 5; thus, we add 1 to (“bump up” by 1) the 9, making 10.
3. Replace each of the three digits to the right of the 10 with a 0 to keep the number 10 in the thousands place: 10,000

Answer: The estimate for 9,502 is 10,000.

<table>
<thead>
<tr>
<th>Practice 9: Estimate 961</th>
<th>Answer: 1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watch It: <a href="http://youtu.be/c2v9pSJQaz">http://youtu.be/c2v9pSJQaz</a></td>
<td></td>
</tr>
</tbody>
</table>

**Example 10:** Estimate 0.00235

1. The left-most non-zero digit is the 2 in the thousandths place.
2. The digit immediately to its right is 3, which is less than 5; thus, the 2 remains unchanged.
3. The digit 2 is in the thousandths place. Therefore we do not need zeros to the right of the 2: 0.002

Answer: The estimate for 0.00235 is 0.002.

<table>
<thead>
<tr>
<th>Practice 10: Estimate 0.0649</th>
<th>Answer: 0.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Watch It: <a href="http://youtu.be/V4mth6lE2Q">http://youtu.be/V4mth6lE2Q</a></td>
<td></td>
</tr>
</tbody>
</table>

**Rounding Decimals**

If you buy a pizza that costs $12.38, you could say that it costs you about $12. You would then expect to receive about $8 in change from a $20 bill. In this case, we are not estimating; instead we are rounding $12.38 to the nearest whole dollar to get $12.00. (The “estimate” for $12.38, according to the rules above, would be $10.00).

To round to the nearest dollar (that is, to the ones place), we look at the 2 in the ones place and then look at the digit immediately to its right, the 3. Since 3 is less than 5, the 2 remains unchanged. Then, as before, each digit to the right of the 2 is replaced with a 0.

Rounding $12.38 to the nearest whole dollar (to the ones place) gives us $12.00 = $12  
Because $12 is less than $12.38, we say that we have “rounded down.”

Suppose that we want to round $4.375 to the nearest penny. We would have to look at the 7 in the hundredths place (each hundredth of a dollar is a penny) and then look at the digit 5 immediately to its right. Since that 5 is 5 or more, we add 1 to (“bump up” by 1) the 7, making an 8. We also replace each digit to the right of the 8 with a 0.

Rounding $4.375 to the nearest penny (to the nearest hundredth) gives us $4.38. Because $4.38 is more than $4.375, we say that we have “rounded up.”
IMPORTANT NOTE: Notice that we do not move the decimal point when we round. For example, rounding to two decimal places does not mean that we move the decimal point two places; it means that the answer will have two digits to the right of the decimal point.

We can introduce some notation that is a helpful tool in the rounding process.

### Rounding Decimals

1. Identify the digit that you are rounding by circling it or pointing to it.

2. Place a vertical line immediately to the right of the digit.

3. Look at the digit immediately to the right of the vertical line to determine if you round up or leave unchanged.

4. If needed, write zeros to keep the remaining digits in their original place.

Let’s take another look at rounding 4.375 to the nearest hundredth:

1. Identify the digit that you are rounding by circling it or pointing to it: 4.37\(\uparrow\)5
2. Place a vertical line immediately to the right of that digit: 4.37|5
3. The digit to the right of the vertical line is what we use to determine whether we round up or leave unchanged.
4. We do not need to write zeros because the remaining digits are in their original place value: 4.38|0

Answer: 4.375 rounded to the nearest hundredth is 4.38.

**Example 11:** Round 523.4187 to the nearest hundredth.

The digit 1 is in the hundredths place. Place a vertical line to right of the 1: 523.41|87
The digit to the right of the vertical line is 8, which is greater than or equal to 5. So, we add 1 to (“bump up” by 1) the 1, making 2.

We do not need to write zeros because the remaining digits are in their original place value: 523.42

Answer: 523.4187 rounded to the nearest hundredth is 523.42.

**Practice 11:** Round 523.4187 to the nearest tenth.  
Answer: 523.4

**Watch It:** [http://youtu.be/4UUmxrPdCKI](http://youtu.be/4UUmxrPdCKI)
**Example 12:** Round 12,345 to the nearest hundred.

The digit 3 is in the hundreds place. Place a vertical line to right of the 3: \[12,3\underline{45}\]

The digit to the right of the vertical line is a 4, which is less than 5. Thus, the 3 remains unchanged.

We also replace each digit to the right of the vertical line with a 0 to keep the remaining digits in their original place value. \[12,3\underline{00}\]

**Answer:** 12,345 rounded to the nearest hundred is 12,300

**Practice 12:** Round 12,345 to the nearest ten. **Answer:** 12,350


**Example 13:** Round 7.96 to the nearest tenth.

The digit 0 is in the tenths place. Place a vertical line to right of the 0: \[7.9\underline{6}\]

The digit to the right of the vertical line is a 6, which is greater than 5. So, we add 1 to (“bump up” by 1) the 9. However, we cannot write a 10 where the 9 is, so we write a 0 in place of the 9 and “carry a 1” (add 1 to) the place immediately to the left of the 9.

\[= 8.0\]

We do not need to write zeros because the remaining digits are in their original place value: \[8.0\]

Notice that we must keep the digit 0 to the left of the vertical line and not write 8.0 as 8. Because we were asked to round to the tenths place, there must be a digit appearing in the tenths place, even if that digit is 0.

**Answer:** 7.96 rounded to the nearest tenth is \[8.0\]

**Practice 13:** Round 5.698 to the nearest hundredth. **Answer:** 5.70

*Watch It:* [http://youtu.be/ASOE7O3L0Ww](http://youtu.be/ASOE7O3L0Ww)
Example 14: Round 17.31958 to three decimal places.

The digit 9 is in the third decimal place. Place a vertical line to right of the 9: \[ 17.319 \overline{58} \]

The digit to the right of the vertical line is a 5. So, we add 1 to (“bump up” by 1) the 9.
However, we cannot write a 10 where the 9 is, so we write a 0 in place of the 9 and “carry a 1” (add 1 to) the place immediately to the left of the 9.

\[ \begin{array}{c|c} \hline & 1 7 . 3 2 0 \\ \hline \end{array} \]

We do not need to write zeros because the remaining digits are in their original place value: 17.320

Note that we keep the 0 to the left of the vertical line, to fulfill the directions to round to 3 decimal places.

Answer: 17.31958 rounded to 3 decimal places is 17.320

Practice 14: Round 5349.8162 to three decimal places.  
Answer: 5349.816

Watch It:  
http://youtu.be/IR4PHXNQcng

Example 15: Round 2.5997 to three decimal places.

The digit 9 is in the third decimal place. Place a vertical line to right of the 9: \[ 2.599 \overline{7} \]

The digit to the right of the vertical line is a 7. So, we add 1 to (“bump up” by 1) the 9.
However, we cannot write a 10 where the 9 is, so we write a 0 in place of the 9 and “carry a 1” (add 1 to) the place immediately to the left of the 9 – which is a 9 itself, so we again write a 0 in that place as well and carry a 1 to the digit immediately to its left.

\[ \begin{array}{c|c} \hline & 2 . 6 0 0 \\ \hline \end{array} \]

We do not need to write zeros because the remaining digits are in their original place value: 2.600

Note that we keep the zeros to the left of the vertical line, to fulfill the directions to round to 3 decimal places.

Answer: 2.5997 rounded to 3 decimal places is 2.600

Practice 15: Round 8.597 to two decimal places.  
Answer: 8.60

Watch It:  
http://youtu.be/4dcrJpPZmeo

Watch All:  
http://youtu.be/W8JTMfDMeqY
4.1 Introduction to Decimals Exercises

1. In the number 642.359, the digit in the…
   a. … tenths place is: __________
   b. … hundredths place is: __________
   c. … thousandths place is: __________
   d. … ones place is: __________
   e. … tens place is: __________
   f. … hundreds place is: __________

2. Consider the number 547.219
   a. Name the place value of 2: __________
   b. Name the place value of 4: __________
   c. Name the place value of 7: __________
   d. Name the place value of 1: __________
   e. Name the place value of 9: __________
   f. Name the place value of 5: __________

3. In any decimal, name the place value.
   a. 2 places to the right of the decimal point: __________
   b. 2 places to the left of the decimal point: __________
   c. 3 places to the right of the decimal point: __________
   d. 1 place to the left of the decimal point: __________
   e. 1 place to the right of the decimal point: __________
4. Compare the decimal numbers by filling in the blank with <, >, or =.
   \[0.701 \quad \underline{\quad} \quad 0.71\]

5. Compare the decimal numbers by filling in the blank with <, >, or =.
   \[51.24 \quad \underline{\quad} \quad 51.2\]

6. Compare the decimal numbers by filling in the blank with <, >, or =.
   \[2.7 \quad \underline{\quad} \quad 2.700\]

7. Write the decimal numbers in order from least to greatest:
   \[4.17 \quad 4.16 \quad 4.162 \quad 4.017\]

8. According to Wikipedia, Ty Cobb’s batting averages for seven years were:
   \[0.350 \quad 0.324 \quad 0.377 \quad 0.420 \quad 0.409 \quad 0.390 \quad 0.368\]
   Find his median batting average for those years.

9. Estimate 289: ___________

10. Estimate 14,567: ___________

11. Estimate 963,146: ___________

12. Estimate 2.8135: ___________

13. Estimate 0.0419: ___________

14. Estimate 0.65: ___________

15. Estimate 0.00372: ___________

16. Round 24.617 to the nearest tenth: ___________

17. Round 341.087 to the nearest hundredth: ___________

18. Round 0.1596 to 3 decimal places: ___________

19. Round 1,206.995 to the nearest hundredth: ___________

20. Round 7.15743 to the nearest ten-thousandth: ___________
4.1 Introduction to Decimals Exercises Answers

1. a. 3
   b. 5
   c. 9
   d. 2
   e. 4
   f. 6

2. a. tenths
   b. tens
   c. ones
   d. hundredths
   e. thousandths
   f. hundreds

3. a. hundredths
   b. tens
   c. thousandths
   d. ones
   e. tenths

4. 0.701 < 0.71
5. 51.24 > 51.2
6. 2.7 = 2.700
7. 4.017, 4.16, 4.162, 4.17
8. 0.377
9. 300
10. 10,000
11. 1,000,000
12. 3
13. 0.04
14. 0.7
15. 0.004
16. 24.6
17. 341.09
18. 0.160
19. 1,207.00
20. 7.1574