5.2 Algebraic Properties

Your friend’s birthday is next weekend, and you are sending her a birthday card. As usual, you will put a return address label in the upper left corner of the envelope and a stamp in the upper right corner. Does the order in which you place these—label then stamp or stamp then label—on the envelope matter? No, the end result will be the same regardless of which you put on first and which you put on second.

Now you are ready to go out to mail the card, but you need to put your socks and shoes on. Does the order in which you place these on your feet matter? I think so! It may not work out so well if you place your shoes on first, then your socks!

The point is that sometimes the order in which we perform tasks matters and sometimes it does not. How about in math? Does the order in which we perform mathematical operations matter? We begin this section of Chapter 5 by investigating that question and stating our results in the form of Algebra Properties. In fact there are five Algebra Properties that will be presented in this section. You will learn to recognize each property and also how to apply each property to make problem solving easier.

Commutative Property of Addition

You have invited some of your friends to get together Friday night for pizza at your house. Your friend Mike volunteered to bring the pizza and your friend Sara volunteered to bring soda. You and your friends agree to split equally the cost of the food and drinks. To do that you need the total amount that Mike and Sara paid. Mike said that the pizza cost $36.89, and Sara said that the soda cost $6.39. When you calculate the total cost, does it matter in which order you write down the two costs? No.

\[
\begin{align*}
\text{Pizza:} & \quad 36.89 \\
\text{Soda:} & \quad 6.39 \\
\text{TOTAL:} & \quad 43.28
\end{align*}
\]

Adding the prices in this order: Pizza Cost + Soda Cost = $43.28.

Adding the prices in this order: Soda Cost + Pizza Cost = $43.28.

So, the order in which the amounts were added did not matter. This is always true in any addition problem regardless of the numbers being added. In algebra, we call this property the Commutative Property of Addition. And since variables represent numbers in algebra, the property applies to all algebraic terms, not just numbers. So, the property simply says that the order in which we add does not matter. We will get the same answer (called the sum) regardless of the order in which we add the terms.
Commutative Property of Multiplication

One way to interpret a multiplication problem such as \( a \cdot b \) is to calculate \( a \) groups of \( b \).

You went to the store and purchased 3 bags of apples with 5 apples in each bag. You have 3 groups of 5 apples. To determine how many apples you bought you would multiply as follows:

\[ 3 \cdot 5 = 15 \text{ apples} \]

Your friend went to a different store and purchased 5 bags of apples with 3 apples in each bag. Your friend has 5 groups of 3 apples. To determine how many apples your friend bought you would multiply as follows:

\[ 5 \cdot 3 = 15 \text{ apples} \]

You and your friend purchased the same number of apples. The way they were packaged was the only difference. Notice that the order in which we multiply factors does not matter. We will get the same answer (called the product) regardless of the order in which we multiply factors.

<table>
<thead>
<tr>
<th>WORDS</th>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changing the order in which two factors are multiplied does not change the product (answer).</td>
<td>( a \cdot b = b \cdot a )</td>
<td>( 2 \cdot 6 = 6 \cdot 2 )</td>
</tr>
</tbody>
</table>
Example 1: Rewrite each expression using the *Commutative Property of Multiplication* or *Addition*.

\[
x + 5 \quad \text{Answer:} \quad 5 + x
\]
\[
(x)(8) \quad \text{Answer:} \quad (8)(x) \text{ which equals } 8x
\]
\[
4 + 9x \quad \text{Answer:} \quad 9x + 4
\]
\[
xy \quad \text{Answer:} \quad xy
\]
\[
2 - 3x \quad \text{Answer:} \quad -3x + 2
\]
\[
-5x + 6 \quad \text{Answer:} \quad 6 + (-5x)
\]
\[
(-2)(3) \quad \text{Answer:} \quad (3)(-2)
\]

Practice 1: Rewrite each expression using the *Commutative Property of Multiplication* or *Addition*.

<table>
<thead>
<tr>
<th></th>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>( y + 9 )</td>
<td>( 9 + y )</td>
</tr>
<tr>
<td>b</td>
<td>((c)(-7))</td>
<td>((-7)(c))</td>
</tr>
<tr>
<td>c</td>
<td>(-3 + 6w)</td>
<td>(6w + (-3))</td>
</tr>
<tr>
<td>d</td>
<td>((a)(c))</td>
<td>((c)(a))</td>
</tr>
<tr>
<td>e</td>
<td>(4x - 5 = 4x + (-5))</td>
<td>(-5 + 4x)</td>
</tr>
</tbody>
</table>

Answers: a. \( y + 9 \)  \hspace{1cm} b. \((-7)(c)\)  \hspace{1cm} c. \(6w + (-3)\)  \hspace{1cm} d. \((c)(a)\)  \hspace{1cm} e. \(-5 + 4x\)

Watch It: [http://youtu.be/QrzNBkUQcYI](http://youtu.be/QrzNBkUQcYI)

The real beauty and power of the *Commutative Property of Multiplication* is seen when we multiply three or more factors. For instance, to solve the problem \(5 \cdot 37 \cdot 2\), we could follow our typical order of operations and work left to right.

First we must multiply 5 with 37. Take a moment to perform that multiplication.

\[= 5 \cdot 37 \cdot 2\]
\[= 185 \cdot 2\]

Next, complete the problem by multiplying 185 with 2.
The final answer is 370.

Did you get that? Wasn’t all that multiplication fun?

Now we are going to rework the exact same problem, but this time, let’s use the *Commutative Property of Multiplication*. The *Commutative Property of Multiplication* says that the order of the factors does not matter.

Switch the order of the second and third numbers.

\[= 5 \cdot 37 \cdot 2\]

Work from left to right just as we did last time.

First multiply 5 with 2. That’s 10, a simple product to figure out.

\[= 5 \cdot 2 \cdot 37\]
\[= 10 \cdot 37\]

Finally, multiplying a whole number by 10 is as simple as placing a 0 at the end of that whole number.

\[= 370\]
So, while our first method involved a mildly time consuming multiplication problem, you must admit that the second method was much quicker and easier. Now do you see the beauty and power of the *Commutative Property of Multiplication*?

**Calculate the problem using the appropriate *Commutative Property*.**

**Example 2:**  \[ 23 + 39 + 17 \]

\[
\begin{align*}
&= 23 + 17 + 39 \\
&= 40 + 39 \\
&= 79
\end{align*}
\]

****Calculate the problem using the appropriate *Commutative Property*.**

**Practice 2:**  \[ 42 + 69 + 18 \]

**Answer:**  \[ 42 + 18 + 69 = 60 + 69 = 129 \]

**Watch It:**  [http://youtu.be/_JKdgpP0sd4](http://youtu.be/_JKdgpP0sd4)

**Calculate the problem using the appropriate *Commutative Property*.**

**Example 3:**  \[ 4 \cdot 9 \cdot 5 \]

\[
\begin{align*}
&= 4 \cdot 5 \cdot 9 \\
&= 20 \cdot 9 \\
&= 180
\end{align*}
\]

***Calculate the problem using the appropriate *Commutative Property*.***

**Practice 3:**  \[ 5 \cdot 14 \cdot 2 \]

**Answer:**  \[ 5 \cdot 2 \cdot 14 = 10 \cdot 14 = 140 \]

**Watch It:**  [http://youtu.be/CacIX36_0SI](http://youtu.be/CacIX36_0SI)

**Simplify the problem using the appropriate *Commutative Property*, then combine like terms.***

**Example 4:**  \[ 6x + 12 + 14x \]

\[
\begin{align*}
&= 6x + 14x + 12 \\
&= 20x + 12
\end{align*}
\]

Using the *Commutative Property of Addition*, *rewrite* the expression with the *like terms* next to each other.

Combine *like terms*.

**Simplify the problem using the appropriate *Commutative Property*, then combine like terms.**

**Practice 4:**  \[ 5a – 83 – 13a \]

**Answer:**  \[ -8a – 83 \]

**Watch It:**  [http://youtu.be/x0XYfNVcU-E](http://youtu.be/x0XYfNVcU-E)
Simplify the problem using the appropriate *Commutative Property*, then combine like terms.

**Example 5:** \(3x + 4y - 10x + 7y\)

\[
= 3x + 4y + (-10x) + 7y \quad \text{Write as a sum of terms.}
\]

\[
= 3x + (-10x) + 4y + 7y \quad \text{Using the *Commutative Property of Addition*,}
\]

\[
= -7x + 11y \quad \text{Combine like terms.}
\]

**Note:** The sign (+/-) to the left of a term is part of the term and thus will move with the term when using the *Commutative Property*.

---

**Simplify the problem using the appropriate *Commutative Property*, then combine like terms.**

**Practice 5:** \(8x - 3y + 2x - 5y\) **Answer:** \(10x - 8y\)

**Watch It:** [http://youtu.be/PwanGdXLM74](http://youtu.be/PwanGdXLM74)

---

**Simplify the problem using the appropriate *Commutative Property*, then combine like terms.**

**Example 6:** \(-2x + 4 - 7 + 8x + 11x - 6\)

\[
=(-2x) + 4 + (-7) + 8x + 11x + (-6) \quad \text{Write as a sum of terms.}
\]

\[
= -2x + 8x + 11x + 4 + (-7) + (-6) \quad \text{Using the *Commutative Property of Addition*,}
\]

\[
= 17x - 9 \quad \text{Combine like terms.}
\]

---

**Simplify the problem using the appropriate *Commutative Property*, then combine like terms.**

**Practice 6:** \(-7a + 3 + 1 + a + 4a - 5\) **Answer:** \(-2a - 1\)

**Watch It:** [http://youtu.be/1agmTDfnO1g](http://youtu.be/1agmTDfnO1g)
Associative Property of Addition

George and Angela work at the CCBC bookstore and are preparing to stock the shelves before the semester starts. In the stock room, George counts the number of Calculus 1 books and puts them in a box. Then he counts the number of Calculus 2 books and puts them in the same box. Then he counts the number of Calculus 3 books, but puts them in a second box since the first box is full. Before going on break, George calculates the total number of Calculus books as shown below and records his total.

![Diagram showing the calculation process](Image from Microsoft Office Clip Art)
While George is on break, Angela begins to pick up the first box of Calculus books to carry it out and place the books on the shelf. But she finds it much too heavy to pick up. She notices that there aren’t many books in the second box. So Angela takes the Calculus 2 books out of the first box and places them in the second box to distribute the weight more evenly. She then carries each box out. Angela takes the Calculus 1 books out of the first box, counts them, and places them on the shelf. She then takes the Calculus 2 books and the Calculus 3 books out of the second box, counts them, and places them on the shelf. Angela counts the books as shown below and records her total.

George obtained the total number of books by adding: \((40 + 30) + 15\).

Angela obtained the total number of books by adding: \(40 + (30 + 15)\).

Notice that George and Angela both had the same total of 85 books, and that is exactly what we would expect. Pay particular attention to how the two methods differ. It is NOT the order of the numbers that differs. The order was the same in both methods. Both George and Angela started with the number 40, then followed with the number 30, and ended with the number 15. What IS different is the grouping of the numbers. George grouped 40 and 30 together in parentheses because he had the Calculus I and II books in the same box. Angela, on the other hand, grouped the Calculus II and III books in the same box, so she grouped the numbers 30 and 15 together in parentheses.

So, the grouping of the numbers did not matter in the addition problem. This is always true in any addition problem regardless of the numbers being added. In algebra, we call this property
the Associative Property of Addition. As with the Commutative Property of Addition, it applies to all algebraic terms, not just numbers. So, the property simply says that the grouping in an addition problem does not matter. We will get the same answer (called the sum) regardless of how the terms are grouped. When you use the Associative Property of Addition, think of the word “association” which means “group.” This will help you to remember that the property involves a change in grouping.

### ASSOCIATIVE PROPERTY OF ADDITION

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<td>Changing the <strong>grouping</strong> of terms being added does not change the sum (answer).</td>
<td>((a + b) + c = a + (b + c))</td>
<td></td>
</tr>
<tr>
<td>Notice that the order of the terms is the same on both sides of the equation.</td>
<td>((4 + 6) + 2 = 4 + (6 + 2))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10 + 2 = 4 + 8)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(12 = 12)</td>
<td></td>
</tr>
</tbody>
</table>

There is an Associative Property of Multiplication as well. The grouping of the factors in a multiplication problem does not matter. We will get the same answer (called the product) regardless of how the factors are grouped.

### ASSOCIATIVE PROPERTY OF MULTIPLICATION

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Changing the <strong>grouping</strong> of factors being multiplied does not change the product (answer).</td>
<td>((ab)c = a(bc))</td>
<td></td>
</tr>
<tr>
<td>Notice that the order of the factors is the same on both sides of the equation.</td>
<td>((4 \cdot 6) \cdot 2 = 4 \cdot (6 \cdot 2))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(24 \cdot 2 = 4 \cdot 12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(48 = 48)</td>
<td></td>
</tr>
</tbody>
</table>
Rewrite each expression using the appropriate Associative Property.

**Example 7:** \((7 + x) + 8\)  
Answer: \(7 + (x + 8)\)

Note: The order of the terms is the same in the original expression as it is in the answer. What has changed is the grouping. The 7 and \(x\) are grouped together in the original expression, whereas the \(x\) and 8 are grouped together in the answer.

**Practice 7:** \((y + 8) + 4\)  
Answer: \(y + (8 + 4)\)

**Watch It:** [http://youtu.be/mIGmNxrjnjU](http://youtu.be/mIGmNxrjnjU)

**Example 8:** \(3 \cdot (9 \cdot y)\)  
Answer: \((3 \cdot 9) \cdot y\)

Note: The order of the factors is the same in the original expression as it is in the answer. What has changed is the grouping. The 9 and \(y\) are grouped together in the original expression, whereas the 3 and 9 are grouped together in the answer.

**Practice 8:** \(5 \cdot (7 \cdot a)\)  
Answer: \((5 \cdot 7) \cdot a\)

**Watch It:** [http://youtu.be/iqy9iLp7VTs](http://youtu.be/iqy9iLp7VTs)

Show that each statement is true using the appropriate Associative Property.

**Example 9:** \((6 + 2) + 5 = 6 + (2 + 5)\)

\[
(6 + 2) + 5 = 6 + (2 + 5) \\
8 + 5 = 6 + 7 \\
13 = 13
\]

**Practice 9:** \((7 + 4) + 8 = 7 + (4 + 8)\)  
Answer: \(19 = 19\)

**Watch It:** [http://youtu.be/UefDZsxONvQ](http://youtu.be/UefDZsxONvQ)

**Example 10:** \(4 \cdot (2 \cdot 3) = (4 \cdot 2) \cdot 3\)

\[
4 \cdot (2 \cdot 3) = (4 \cdot 2) \cdot 3 \\
4 \times 6 = 8 \times 3 \\
24 = 24
\]

**Practice 10:** \(2 \cdot (7 \cdot 9) = (2 \cdot 7) \cdot 9\)  
Answer: \(126 = 126\)

**Watch It:** [http://youtu.be/9b4X5CTLj3E](http://youtu.be/9b4X5CTLj3E)
We demonstrated the power and beauty of the *Commutative Property* just a few pages back. Let’s illustrate the power and beauty of the *Associative Property* with the problem \( \left( \frac{1}{3} + \frac{3}{8} \right) + \frac{5}{8} \).

To make this computation, we follow the order of operations discussed in Chapter 1.

We start by adding the fractions in the parentheses.

\[
\left( \frac{1}{3} + \frac{3}{8} \right) + \frac{5}{8}
\]

To add fractions you need a Least Common Denominator. The Least Common Denominator is 24.

\[
\left( \frac{8}{24} + \frac{9}{24} \right) + \frac{5}{8}
\]

Perform the pair of multiplications inside the parentheses.

The two fractions inside the parentheses have a common denominator. So, we add those fractions.

\[
\frac{17}{24} + \frac{5}{8}
\]

To add these two fractions, we need a common denominator. The Least Common Denominator, once again, is 24.

\[
\frac{17}{24} + \frac{5}{8} \cdot \frac{3}{3}
\]

Perform the multiplication.

\[
\frac{17}{24} + \frac{15}{24}
\]

Perform the addition.

\[
\frac{32}{24}
\]

Simplify.

\[
\frac{32}{24} = \frac{4}{3}
\]

Convert to a mixed fraction.

\[
1 \frac{1}{3}
\]
Hopefully, you haven’t lost sight of our purpose here. We are trying to show the usefulness of the **Associative Property**. Next, we will rework the same problem after applying the **Associative Property of Addition**.

Compute by using the **Associative Property of Addition**.

\[
\left( \frac{1}{3} + \frac{3}{8} \right) + \frac{5}{8}
\]

First apply the **Associative Property of Addition** in order to regroup the terms.

\[
= \frac{1}{3} + \left( \frac{3}{8} + \frac{5}{8} \right)
\]

Following the order of operations, we add the fractions inside the parentheses. This is easy to do because they have the same denominator.

\[
= \frac{1}{3} + \frac{8}{8}
\]

A fraction with the same numerator and denominator is 1.

\[
= \frac{1}{3} + 1
\]

Write answer as a mixed number

\[
= \frac{1}{3}
\]

What a difference in these two methods! The first method took 9 steps and quite a bit of time and work. The second method took only 4 steps and hardly any time or work. So, are you now impressed by the power of the **Associative Property**? As shown in some of the examples below, the property is also very useful in simplifying algebraic expressions in order to get them in proper form and make them easier to work with.
Simplify each expression using the appropriate **Associative Property**.

**Example 11:**  
\[(x + 3) + 6\]  
\[= x + (3 + 6)\]  
Regroup *like terms* using **Associative Property of Addition**.  
\[= x + 9\]  
Simplify inside the parentheses.

**Practice 11:**  
\[(y + 8) + 4\]  
**Answer:**  
\[y + 12\]  
**Watch It:**  
[http://youtu.be/f1DnP6z1DIw](http://youtu.be/f1DnP6z1DIw)

**Example 12:**  
\[8(4x)\]  
\[= 8 \cdot (4 \cdot x)\]  
\[= (8 \cdot 4) \cdot x\]  
Regroup using **Associative Property of Multiplication**.  
\[= 32x\]  
Simplify inside the parentheses.

**Practice 12:**  
\[5 \cdot (7 \cdot a)\]  
**Answer:**  
\[35a\]  
**Watch It:**  
[http://youtu.be/0iOS1KOshN0](http://youtu.be/0iOS1KOshN0)

**Example 13:**  
\[5x + (2x + 10)\]  
\[= (5x + 2x) + 10\]  
Regroup *like terms* using **Associative Property of Addition**.  
\[= 7x + 10\]  
Simplify inside the parentheses by combining *like terms*.

**Practice 13:**  
\[9a + (4a + 6)\]  
**Answer:**  
\[13a + 6\]  
**Watch It:**  

**Example 14:**  
\[12x + \left(3 \frac{1}{2}\right) + 4 \frac{1}{2}\]  
\[= 12x + \left(3 \frac{1}{2} + 4 \frac{1}{2}\right)\]  
Regroup using the **Associative Property of Addition**.  
\[= 12x + \left(3 + \frac{1}{2} + 4 + \frac{1}{2}\right)\]  
Write mixed numbers using addition.  
\[= 12x + \left(3 + 4 + \frac{1}{2} + \frac{1}{2}\right)\]  
Rewrite using **Commutative Property of Addition**.  
\[= 12x + \left(7 + \frac{2}{2}\right)\]  
Add whole numbers together and fractions together.  
\[= 12x + (7 + 1)\]  
Simplify.  
\[= 12x + 8\]
**Practice 14:** \[
\left(15x + 7 \frac{1}{3}\right) + 4 \frac{2}{3}
\]

**Answer:** \(15x + 12\)

**Watch It:** [http://youtu.be/aehQg4rJLaG](http://youtu.be/aehQg4rJLaG)

Note: The operations of subtraction and division do not have a *Commutative Property* or an *Associate Property*.

Subtraction is NOT commutative:

\[
8 - 5 \neq 5 - 8
3 \neq -3
\]

Subtraction is NOT associative:

\[
100 - (30 - 10) \neq (100 - 30) - 10
100 - 20 \neq 70 - 10
80 \neq 60
\]

Division is NOT commutative:

\[
12 \div 4 \neq 4 \div 12
\]

Division is NOT associative:

\[
64 \div (8 \div 4) \neq (64 \div 8) \div 4
64 \div 2 \neq 8 \div 4
32 \neq 2
\]
Identity Property

Nick is a dialysis patient. Each time he goes to the dialysis center for treatment, the nurse begins by weighing Nick. It is very important to compare Nick’s previous weight with his current weight in order to determine how much fluid to take off during the dialysis treatment. This time, the nurse weighs Nick and records the following in her chart.

<table>
<thead>
<tr>
<th>Date</th>
<th>Previous Weight</th>
<th>Weight Gain/Loss</th>
<th>Current Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>08/29</td>
<td>140</td>
<td>0</td>
<td>140</td>
</tr>
</tbody>
</table>

Since Nick had no weight gain this time, his current weight is 140 + 0, which, of course, equals 140. Thus, Nick’s current weight is the same as his previous weight. In any addition problem, adding 0 to a number results in the same number. In algebra, this fact is called the Identity Property of Addition. When you use the Identity Property of Addition, think of the word “identical.” Adding 0 to a term produces the identical term.

<table>
<thead>
<tr>
<th>WORDS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>The sum of any number and 0 is the same number.</td>
<td>(a + 0 = a)</td>
<td>(6 + 0 = 0 + 6 = 6)</td>
</tr>
<tr>
<td>The sum of any term and 0 is the same term.</td>
<td>(a + 0 = a)</td>
<td>(2x + 0 = 0 + 2x = 2x)</td>
</tr>
</tbody>
</table>

Note: The Additive Identity Element is 0.

There is also an Identity Property of Multiplication. Does it work exactly the same? For instance, does \(6(0) = 6\)? No, we know that \(6(0) = 0\). So, 0 is not the multiplicative identity.

The question is: \(6 \cdot \Box = 6\)? Hopefully you see that the number 1 must fill in the box to make a true statement. Multiplying a number or expression by 1 results in that same number or expression. That is exactly what the Identity Property of Multiplication states.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>The product of any factor and 1 is that same factor.</td>
<td>(a \cdot 1 = a)</td>
<td>(6 \cdot 1 = 1 \cdot 6 = 6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2x \cdot 1 = 1 \cdot 2x = 2x)</td>
</tr>
</tbody>
</table>

Note: The Multiplicative Identity Element is 1.
Example 15: Complete each equation using the appropriate Identity Property.

\[ 7x + 0 = ? \]  
Answer: \( 7x + 0 = 7x \)

\[ 5x(1) = ? \]  
Answer: \( 5x(1) = 5x \)

\[ 0 + ? = 9x \]  
Answer: \( 0 + 9x = 9x \)

\[ 2x(?) = 2x \]  
Answer: \( 2x(1) = 2x \)

Practice 15: Complete each equation using the appropriate Identity Property.

a. \( 8a + 0 = ? \)
b. \( 9y(1) = ? \)
c. \( 0 + ? = 13w \)
d. \( 6a(?) = 6a \)

Answers: a. \( 8a \)  b. \( 9y \)  c. \( 13w \)  d. \( 1 \)

Watch It:  [http://youtu.be/mLhlbiASIvE](http://youtu.be/mLhlbiASIvE)

Inverse Property of Addition

The Student Government Association is sponsoring a dance, spending $624 on a DJ, decorations, and refreshments. How much money does the SGA need to make from ticket sales in order to break even—that is, to pay all of their expenses (without making any profit)? The money the SGA has already spent is represented as a negative number, -624. The goal to break even is represented as the number 0. A positive 624 is needed to offset the -624. Thus, the SGA will need to make $624 from ticket sales in order to break even. This can be written as:

\[ -624 + 624 = 0. \]

In fact, the sum of any number and its opposite is 0. In algebra, we state this as the Inverse Property of Addition.

Moreover, -624 is said to be the opposite of 624, and 624 is said to be the opposite of -624. Another term for opposite is additive inverse.
Inverse Property of Multiplication

There is also an *Inverse Property of Multiplication*. It would be used if we multiplied two numbers together and obtained the answer 1, which is the *multiplicative identity*. The two numbers that multiply together equaling 1 are *reciprocals* of each other. Recall we first learned about reciprocals in chapter 3 when we divided fractions.

For instance, \( \frac{1}{8} \cdot 8 = \frac{1}{8} \cdot 8 = \frac{8}{8} = 1 \).

The number \( \frac{1}{8} \) is the reciprocal of 8, and 8 is the reciprocal of \( \frac{1}{8} \). Another term for *reciprocal* is *multiplicative inverse*.

### INVERSE PROPERTY OF MULTIPLICATION

<table>
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<tr>
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<tbody>
<tr>
<td>The product of a number and its reciprocal is 1. The product of a term and its reciprocal is 1.</td>
<td>( a \cdot \frac{1}{a} = 1 )</td>
<td>( 5 \cdot \frac{1}{5} = 1 ) ( 2x \cdot \frac{1}{2x} = 1 )</td>
</tr>
</tbody>
</table>
Complete each equation using the appropriate Inverse Property.

**Example 16:** Complete each equation using the appropriate Inverse Property.

\[
9 + ? = 0 \quad \text{Answer:} \quad 9 + 9 = 0
\]

\[
2 \cdot ? = 1 \quad \text{Answer:} \quad 2 \cdot \frac{1}{2} = 1
\]

\[
x + (-x) = ? \quad \text{Answer:} \quad x + (-x) = 0
\]

\[-12 + ? = 0 \quad \text{Answer:} \quad -12 + 12 = 0
\]

\[
\frac{1}{4} \cdot ? = 1 \quad \text{Answer:} \quad \frac{1}{4} \cdot 4 = 1
\]

\[-7y + 7y = ? \quad \text{Answer:} \quad -7y + 7y = 0
\]

\[4x \cdot \ ? = 1 \quad \text{Answer:} \quad 4x \cdot \frac{1}{4x} = 1
\]

**Practice 16:** Complete each equation using the appropriate Inverse Property.

a. \( 7 + ? = 0 \)  
   b. \( 3 \cdot ? = 1 \)  
   c. \( w + (-w) = ? \)  
   d. \( -5 + ? = 0 \)  
   e. \( \frac{1}{5} \cdot ? = 1 \)  
   f. \( -3x + 3x = ? \)  
   g. \( 4x \cdot ? = 1 \)

**Answers:**  
a. \(-7\)  
b. \(\frac{1}{3}\)  
c. \(0\)  
d. \(5\)  
e. \(5\)  
f. \(0\)  
g. \(\frac{1}{4x}\)

**Watch It:**  
[http://youtu.be/RHHQx5Bnxnk](http://youtu.be/RHHQx5Bnxnk)

**Watch All:**  
Zero Property of Multiplication

There is one final property to learn. It is called the Zero Property of Multiplication and works only with multiplication. What does 7 \times 0 equal? How about 18 \times 0? How about -624 \times 0? Hopefully, your answer was 0 for all three of these problems. We know that any number multiplied by 0 will give us an answer of 0. The Zero Property of Multiplication states this fact.

**ZERO PROPERTY OF MULTIPLICATION**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of a number and 0 is 0.</td>
<td>(a \cdot 0 = 0 \cdot a = 0)</td>
<td>(4 \cdot 0 = 0 \cdot 4 = 0)</td>
</tr>
<tr>
<td>The product of a term and 0 is 0.</td>
<td>(2x \cdot 0 = 0 \cdot 2x = 0)</td>
<td></td>
</tr>
</tbody>
</table>

You have learned five properties in this section. Before you attempt the practice problems, take some time to review the summary of the properties. Make sure that you understand the explanations of the properties and how they apply to addition and multiplication.

<table>
<thead>
<tr>
<th>PROPERTY</th>
<th>DESCRIPTION</th>
<th>ADDITION</th>
<th>MULTIPLICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>Changing the order does not change the answer.</td>
<td>(a + b = b + a)</td>
<td>(a \cdot b = b \cdot a)</td>
</tr>
<tr>
<td>Associative</td>
<td>Regrouping does not change the answer.</td>
<td>((a + b) + c = a + (b + c))</td>
<td>((a \cdot b) \cdot c = a \cdot (b \cdot c))</td>
</tr>
<tr>
<td>Identity</td>
<td>The sum of any term and 0 is that same term. The product of any factor and 1 is that same factor.</td>
<td>(a + 0 = a) and (0 + a = a)</td>
<td>(a \cdot 1 = a) and (1 \cdot a = a)</td>
</tr>
<tr>
<td>Inverse</td>
<td>The sum of a number and its opposite is 0. The product of a number and its reciprocal is 1.</td>
<td>(a + (-a) = 0)</td>
<td>(\frac{1}{a} = 1)</td>
</tr>
<tr>
<td>Zero</td>
<td>The product of a number and 0 is 0.</td>
<td>(a \cdot 0 = 0) and (0 \cdot a = 0)</td>
<td></td>
</tr>
</tbody>
</table>
5.2 Algebraic Properties Exercises

Match the expression or equation to the correct property it illustrates.

1. 0
   A. Commutative Property of Addition
2. \(\frac{2 \cdot 3}{3 \cdot 2} = 1\)
   B. Commutative Property of Multiplication
3. \(2x + 4 = 4 + 2x\)
   C. Associative Property of Addition
4. \(x(1) = x\)
   D. Associative Property of Multiplication
5. 1
   E. Identity Property of Addition
6. \((4 + 10) + 6 = 4 + (10 + 6)\)
   F. Identity Property of Multiplication
7. \(8 \cdot 0 = 0\)
   G. Additive Identity Element
8. \(7 + (-7) = 0\)
   H. Multiplicative Identity Element
9. \(6 \times 8 = 8 \times 6\)
   I. Inverse Property of Addition
10. \((3 \cdot 9x) \cdot 5x = 3 \cdot (9x \cdot 5x)\)
    J. Inverse Property of Multiplication
11. \(9 + 0 = 9\)
    K. Zero Property of Multiplication

For problems 12 – 15, rewrite each expression using the given property.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. 18 + 5x</td>
<td>Commutative</td>
</tr>
<tr>
<td>13. (x \cdot \frac{1}{2})</td>
<td>Commutative</td>
</tr>
<tr>
<td>14. ((x + y) + z)</td>
<td>Associative</td>
</tr>
<tr>
<td>15. (2x \cdot (9 \cdot y))</td>
<td>Associative</td>
</tr>
</tbody>
</table>
For problems 16 – 25, complete each equation using the given property.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>16. $8a \cdot 1 = \square$</td>
<td>Identity</td>
</tr>
<tr>
<td>17. $0 + \square = 16x$</td>
<td>Identity</td>
</tr>
<tr>
<td>18. $\frac{5}{6}x \cdot 1 = \square$</td>
<td>Identity</td>
</tr>
<tr>
<td>19. $7.2x + 0 = \square$</td>
<td>Identity</td>
</tr>
<tr>
<td>20. $12 \cdot \frac{1}{12} = \square$</td>
<td>Inverse</td>
</tr>
<tr>
<td>21. $25 + \square = 0$</td>
<td>Inverse</td>
</tr>
<tr>
<td>22. $\frac{a}{b} \cdot \square = 1$</td>
<td>Inverse</td>
</tr>
<tr>
<td>23. $2x + (-2x) = \square$</td>
<td>Inverse</td>
</tr>
<tr>
<td>24. $62 \cdot 0 = \square$</td>
<td>Zero</td>
</tr>
<tr>
<td>25. $x \cdot \square = 0$</td>
<td>Zero</td>
</tr>
</tbody>
</table>

For problems 26, and 27, the statements below illustrate the Associative Property. Show that the statements are true.

26. $(5.2 + 3.1) + 7.4 = 5.2 + (3.1 + 7.4)$
27. $20 \cdot (8 \cdot 5) = (20 \cdot 8) \cdot 5$
For problems 28 – 41, simplify each expression using the given property.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>28. $24 + 17 + 6$</td>
<td>Commutative</td>
</tr>
<tr>
<td>29. $5 \cdot 56 \cdot 2$</td>
<td>Commutative</td>
</tr>
<tr>
<td>30. $8 + 3x + 5$</td>
<td>Commutative</td>
</tr>
<tr>
<td>31. $2a + 15 + 23a$</td>
<td>Commutative</td>
</tr>
<tr>
<td>32. $6x + 21 + 8x$</td>
<td>Commutative</td>
</tr>
<tr>
<td>33. $(42x + 14) + 28$</td>
<td>Associative</td>
</tr>
<tr>
<td>34. $3.6x + (7.3x + 28)$</td>
<td>Associative</td>
</tr>
<tr>
<td>35. $5 \cdot \left( \frac{1}{5} \cdot x \right)$</td>
<td>Associative</td>
</tr>
<tr>
<td>36. $(2 \times 4) \times 25$</td>
<td>Associative</td>
</tr>
<tr>
<td>37. $\left( 4 \frac{1}{4} + 7 \frac{1}{2} \right) + 6 \frac{1}{2}$</td>
<td>Associative</td>
</tr>
<tr>
<td>38. $\left( \frac{3}{5} \cdot 4 \right) \cdot \frac{7}{8}$</td>
<td>Associative</td>
</tr>
</tbody>
</table>
5.2 Algebraic Properties Exercises Answers

1. G
2. J
3. A
4. F
5. H
6. C
7. K
8. I
9. B
10. D
11. E

12. $5x + 18$
13. $\frac{1}{2} \cdot x$
14. $x + (y + z)$
15. $(2x \cdot 9) \cdot y$

16. $8a \cdot 1 = 8a$
17. $0 + \left[ \frac{16x}{x} \right] = 16x$
18. $\frac{5}{6} \cdot x \cdot 1 = \frac{5}{6} x$
19. $7.2x + 0 = 7.2x$
20. $12 \cdot \frac{1}{12} = 1$
21. $25 + \left[ \frac{25}{25} \right] = 0$
22. $\frac{a}{b} \cdot \left[ \frac{b}{a} \right] = 1$
23. $2x + (-2x) = 0$
24. $62 \cdot 0 = 0$
25. $x \cdot 0 = 0$

26. $(5.2 + 3.1) + 7.4 = 5.2 + (3.1 + 7.4)$
   $8.3 + 7.4 = 5.2 + 10.5$
   $15.7 = 15.7$

27. $20 \cdot (8 \cdot 5) = (20 \cdot 8) \cdot 5$
   $20 \cdot 40 = 160 \cdot 5$
   $800 = 800$
28. \[24 + 6 + 17 = 30 + 17 = 47\]

34. \[(3.6x + 7.3x) + 28 = 10.9x + 28\]

29. \[5 \cdot 2 \cdot 56 = 10 \cdot 56 = 560\]

35. \[
\left(\frac{5 \cdot 1}{5}\right) x = 1x = x
\]

30. \[3x + 8 + 5 = 3x + 13\]

or \[8 + 5 + 3x = 13 + 3x\]

36. \[2 \cdot (4 \cdot 25) = 2 \cdot 100 = 200\]

31. \[2a + 23a + 15 = 25a + 15\]

37. \[
4 \frac{1}{4} + \left(7 \frac{1}{2} + 6 \frac{1}{2}\right) = 4 \frac{1}{4} + 14 = 18 \frac{1}{4}
\]

32. \[6x + 8x + 21 = 14x + 21\]

38. \[
\frac{3}{5} \left(\frac{4 \cdot 7}{7 \cdot 8}\right) = \frac{3 \cdot 1}{5 \cdot 2} = \frac{3}{10}
\]

33. \[42x + (14 + 28) = 42x + 42\]