6.5 Applications

You are in the grocery store about to pick up some cans of tuna. There is a 6-ounce can for $1.20 and a 10-ounce can for $1.60. Of course you want the best deal. Which do you buy? Hmmm . . . I believe we have a ratio problem to solve!

In this first type of application problem we need to compare two ratios using unit rates.

**APPLICATION: COMPARING UNIT RATES**

1. For each item, write a ratio (rate) as a fraction.
2. Get the unit rate for each fraction by dividing the numerator by the denominator.
3. Compare the unit rates.

**Example 1:** A 6-ounce can of tuna sells for $1.20 and a 10-ounce can sells for $1.60. Which is the better deal?

\[
\text{Rate} = \frac{\text{Cost}}{\text{# of ounces}} = \frac{\$1.20}{6 \text{ ounces}} \quad \text{and} \quad \frac{\$1.60}{10 \text{ ounces}}
\]

\[
\text{Unit Rate} = \frac{\text{Cost}}{1 \text{ ounce}} = \frac{\$1.20 \div 6 \text{ ounces}}{= \$0.20 \text{ per ounce}} \quad \text{and} \quad \frac{\$1.60 \div 10 \text{ ounces}}{= \$0.16 \text{ per ounce}}
\]

The 10-ounce can of tuna is the better deal because its unit price is lower.

**Practice 1:** A 20-ounce box of cereal costs $4.20 and a 16-ounce box of the same cereal costs $3.60. Which is the better deal?

**Watch It:** [http://youtu.be/z6WusE59AQ](http://youtu.be/z6WusE59AQ) **Answer:** 20-ounce box for $4.20

You are still in the grocery store and you are ready to purchase eggs. You are going to make cupcakes for a family reunion. Your recipe, that makes 18 cupcakes, requires 3 eggs. But you decide to make 4 dozen cupcakes – this is going to be a large reunion. How many eggs will you need? Looks like we have a proportion problem that needs to be solved!

In this type of application, we are dealing with quantities that are proportional. The number of cupcakes you bake is proportional to the number of eggs you use. For instance, if we wanted to double the number of cupcakes and make 36, then we would have to double the number of eggs and use 6. If we wanted to triple the number of cupcakes and make 54, then we would have to triple the number of eggs and use 9. The numbers are not so easy to calculate mentally for 4 dozen, or 48 cupcakes. So we will use algebra. Review the procedure that follows.
APPLICATION: SOLVING PROPORTIONS

1. Assign a variable to the unknown quantity.
2. Set up a ratio (fraction) in words to specify the quantities being compared.
   Use this setup to complete the next two steps.
3. Write a ratio (fraction) using the given values.
4. Write another ratio (fraction) using the unknown quantity (the variable).
5. Write a proportion by setting the ratios equal to each other.
6. Solve the proportion to isolate the variable.

Example 2: A recipe that makes 18 cupcakes requires 3 eggs. How many eggs are needed to make 4 dozen, or 48, cupcakes?

1. **Variable**: assign variable to the unknown quantity $n = \text{number of eggs to make 4 dozen cupcakes}$
2. **Ratio in Words**: quantities being compared $\frac{\text{# of Cupcakes}}{\text{# of Eggs}}$
3. **Ratio of Given Values**: set up like last step $\frac{18 \text{ cupcakes}}{3 \text{ eggs}}$
4. **Ratio with Variable**: set up like last step $\frac{48 \text{ cupcakes}}{n \text{ eggs}}$

   Note that in all three ratios above, cupcakes were in the numerator of the fraction and eggs in the denominator. It is very important to be consistent when you set up your fractions.

5. **Proportion**: set ratios equal $\frac{18}{3} = \frac{48}{n}$
6. **Solve Proportion**: cross multiply $18n = 144$
   
   divide by coefficient of variable $\frac{18n}{18} = \frac{144}{18}$
   
   simplify $n = 8$

You will need 8 eggs to bake 48 cupcakes.
Practice 2:  To make a cup of hot cocoa, Bob mixes 3 teaspoons of cocoa powder with 2 cups of milk. How much cocoa powder would be needed to mix with 12 cups of milk?  

Watch It:  http://youtu.be/TAfbxZ6j24o  

Answer:  18 teaspoons

Example 3:  Water is being pumped out of a basement at a rate of 140 gallons per hour. How many hours will it take to pump 2030 gallons of water out of the basement?  

1. Variable:  assign variable to the unknown quantity  
   \[ n = \text{number of hours to pump water out of basement} \]

2. Ratio in Words:  quantities being compared  
   \[ \frac{\text{gallons of Water}}{\# \text{ of Hours}} \]

3. Ratio of Given Values:  set up like last step  
   \[ \frac{140 \text{ gallons}}{1 \text{ hour}} \]

4. Ratio with Variable:  set up like last step  
   \[ \frac{2030 \text{ gallons}}{n \text{ hours}} \]

   Note that in all three ratios above, the number of gallons of water was in the numerator and the number of hours was in the denominator. It is very important to be consistent with this.

5. Proportion:  set ratios equal  
   \[ \frac{140 \text{ gallons}}{1 \text{ hour}} = \frac{2030 \text{ gallons}}{n \text{ hours}} \]

6. Solve Proportion:  cross multiply  
   \[ 140n = 2030 \]

   divide by coefficient of variable  
   \[ \frac{140n}{140} = \frac{2030}{140} \]

   simplify  
   \[ n = 14.5 \]

The answer represents number of hours, so remember to include those units in your answer.

It will take 14.5 hours to pump 2030 gallons of water from the basement.

Practice 3:  Water is pumped into a pool at a rate of 120 gallons per hour. How many hours will it take to pump 3000 gallons of water into the pool?  

Watch It:  http://youtu.be/T_af_ANj61k  

Answer:  25 hours
Example 4: You need to combine 98 grams of sulfuric acid and 70 grams of sodium hydroxide to produce sodium sulfate (a kind of chemical salt). How many grams of sulfuric acid would need to combine with 20 grams of sodium hydroxide to produce sodium sulfate?

1. Variable: assign variable to the unknown quantity  \( n = \text{grams of sulfuric acid} \) to combine with 20 grams of sodium hydroxide.

2. Ratio in Words: quantities being compared
   \[
   \frac{\text{grams of Sulfuric Acid}}{\text{grams of Sodium Hydroxide}}
   \]

3. Ratio of Given Values: set up like last step
   \[
   \frac{98 \text{ grams Sulfuric Acid}}{70 \text{ grams Sodium Hydroxide}}
   \]

4. Ratio with Variable: set up like last step
   \[
   \frac{n \text{ grams Sulfuric Acid}}{20 \text{ grams Sodium Hydroxide}}
   \]
   Note that in all three ratios above, Sulfuric Acid was in the numerator and Sodium Hydroxide was in the denominator. It is very important to be consistent with this.

5. Proportion: set ratios equal
   \[
   \frac{98}{70} = \frac{n}{20}
   \]

6. Solve Proportion: cross multiply
   \[
   1960 = 70n
   \]
   divide by coefficient of variable
   \[
   \frac{1960}{70} = \frac{70n}{70}
   \]
   simplify
   \[
   28 = n
   \]

The answer represents the number of grams, so remember to include those units in your answer.

We found that 28 grams of Sulfuric Acid are needed to combine with 20 grams of Sodium Hydroxide.

Practice 4: You need to combine 98 grams of sulfuric acid (H2SO4) and 80 grams of sodium hydroxide (NaOH) to produce sodium sulfate (a kind of chemical salt). How many grams of sulfuric acid (H2SO4) would you need to combine with 40 grams of sodium hydroxide (NaOH) to produce sodium sulfate?

Watch It: [https://youtu.be/hTPyyJquW9w](https://youtu.be/hTPyyJquW9w)  Answer: 49 grams
Example 5: You know that there are 56 milligrams of cholesterol in $3\frac{1}{2}$ ounces of trout. How much cholesterol is there in 8 ounces of trout?

1. **Variable**: assign variable to the unknown quantity
   \[ n = \text{mg of cholesterol} \]

2. **Ratio in Words**: quantities being compared
   \[ \frac{\text{mg of Cholesterol}}{\text{ounces of Trout}} \]

3. **Ratio of Given Values**: set up like last step
   \[ \frac{56 \text{ mg of Cholesterol}}{3\frac{1}{2} \text{ ounces of Trout}} \]

4. **Ratio with Variable**: set up like last step
   \[ \frac{n \text{ mg of Cholesterol}}{8 \text{ ounces of Trout}} \]

   Note that in all three ratios above, cholesterol was in the numerator and trout in the denominator. It is very important to be consistent with this.

5. **Proportion**: set ratios equal
   \[ \frac{56}{3\frac{1}{2}} = \frac{n}{8} \]

6. **Solve Proportion**: write mixed number as an improper fraction
   \[ \frac{56}{7} = \frac{n}{8} \]

   cross multiply
   \[ 448 = \frac{7n}{2} \]

   divide by coefficient of variable
   \[ 448 \div \frac{7}{2} = n \]

   simplify
   \[ 448 \div \frac{7}{2} = n \]

   multiply by reciprocal
   \[ \frac{448 \times 2}{1 \times \frac{7}{2}} = n \]

   cancel
   \[ \frac{448 \times \frac{64}{1}}{1 \times \frac{2}{1}} = n \]

   simplify
   \[ 128 = n \]

The answer represents the number of milligrams, so remember to include those units in your answer. There are 128 mg of cholesterol in 8 ounces of trout.
Practice 5: A single tablet of One-A-Day Vitamin for men contains 75 milligrams of Vitamin C. How many milligrams of Vitamin C are in $2 \frac{1}{3}$ tablets of One-A-Day Vitamin?

Watch It: [http://youtu.be/9U8W4iV8gg8](http://youtu.be/9U8W4iV8gg8)  
Answer: 175 milligrams

Watch All: [https://youtu.be/UHU8FtLTH4E](https://youtu.be/UHU8FtLTH4E)
6.5 Application Exercises

Translate each into a proportion and solve the problem.

1. Harry gets 23 mi per gallon of gasoline in his truck. If he has 4 gallons of gasoline in his truck, how far can he go?

2. On a map, 3 in. represents 8 mi. How many in. will represent a distance of 24 mi?

3. On a map, 2 cm represents 3 km. How many km are represented by 15 cm?

4. A nurse has to give a patient a dose of medication. The dosage says 3 ml of medication for a 150 lb. person. If the person weighs 200 lbs., how many ml of medication is the person to receive?

5. 4 mg of a drug are to be given for every 10 kg of body weight. Find the size of a person requiring 25 mg of the drug.

6. A nurse has to give a child a dose of Tylenol. The dosage says 2 tsp of medication for a 50 lb person. If the person weighs 75 lbs, how many tsp of medication will the person receive?

7. A 2-lb box of sugar costs $1.20 and a 5-lb bag of sugar costs $2.75. What is the better deal?

8. A baker can make 72 cookies using 4 c of flour. How many cups of flour are needed to make 288 cookies?

9. Three oz of a chemical are needed to treat 25 oz of water. How many oz of the chemical are needed to prepare 100 oz of water?

10. To make 4 moles of water, 2 moles of oxygen gas are needed. How many moles of water can you make with 21 moles of oxygen gas?
11. A child care advertises that they have a ratio of 2 care givers for every 9 children. If there are 6 care givers, how many children are at the child care?

12. A college has a ratio of 2 male students for every 3 female students. If there are 5322 male students, how many female students attend the college?

13. An office assistant can type 525 words in 5 minutes. At this rate, how many words can the office assistant type in 20 minutes?

14. A baseball player gets 36 hits in 90 times at bat. At this rate, how many hits should he get in 250 times at bat?

15. There are 45 mg of cholesterol in 2 oz of egg substitute. How many mg of cholesterol are there in 3 oz of egg substitute?

16. There are 18 g of fat in a 4 oz steak. How many grams of fat are in a 6 oz steak?

17. At the grocery store, a 16 oz bag of rice is $2.34 and a 24 oz bag of rice is $3.42. What is the better buy?

18. At a warehouse store you can purchase 60 cans of soda for $8.95. At a regular grocery store, you can purchase 12 cans of soda for $1.85. What is the better deal?

19. If 5 snakes can eat 3 mice, how many mice will 30 snakes eat?

20. A recipe calls for \( \frac{1}{4} \) tsp of salt for every \( \frac{1}{2} \) cup of flour. How much salt should be used for 5 cups of flour?
6.5 Application Exercise Answers

1. 92 mi
2. 9 in
3. 22.5 km
4. 4 ml
5. 62.5 kg
6. 3 tsp
7. 5-lb bag
8. 16 c
9. 12 oz of chemical
10. 42 moles of water
11. 27 children
12. 7983 females
13. 2100 words
14. 100 hits
15. 67.5 mg
16. 27 g
17. 24-oz bag
18. 60 cans
19. 18 mice
20. $2\frac{1}{2}$ tsp
### Chapter 6 Summary
Ratios, Rates, & Proportions

#### Section 6.1
**Ratio:** a comparison of two quantities

Write the ratio of 1 cm to 18 cm in 3 ways.  
1. 1 to 18  
2. 1:18  
3. \( \frac{1}{18} \)  

- Always reduce to lowest terms.  
- Never write as a whole number or mixed number.

There are 8 blue cars and 20 green cars on the lot.

- a. Write the ratio of blue cars to green cars.  
  \[
  \frac{8}{20} = \frac{8 \div 4}{20 \div 4} = \frac{2}{5}
  \]

- b. Write the ratio of green cars to the total number of cars.  
  \[
  \frac{20}{28} = \frac{20 \div 4}{28 \div 4} = \frac{5}{7}
  \]

#### Section 6.2
**Rate:** a ratio that compares two quantities that have different units

The patient weighs 150 pounds and receives 45 mg of medicine.  What is the rate of mg to pounds for this medication?  
\[
\frac{45 \text{ mg}}{150 \text{ pounds}} = \frac{45 \div 15 \text{ mg}}{150 \div 15 \text{ pounds}} = \frac{3 \text{ mg}}{10 \text{ pounds}}
\]

#### Section 6.3
**Unit Rate:** a rate in which the denominator is 1

What is the unit rate for driving 126 miles on 4 gallons of gas?  
* Divide numerator and denominator by the denominator:  
\[
\frac{126 \text{ miles}}{4 \text{ gallons}} = \frac{126 \div 4 \text{ miles}}{4 \div 4 \text{ gallons}} = \frac{31.5 \text{ miles}}{1 \text{ gallon}}
\]

#### Section 6.4
**Proportion:** a statement that two ratios (fractions) are equal;  
\[
\frac{a}{b} = \frac{c}{d} \text{ only if } ad = bc
\]

Is \( \frac{3.6}{8} = \frac{4}{9.5} \) a true statement?  
Get cross products:  
\[
(3.6)(9.5) = (8)(4)
\]

The proportion is not true:  
\( 34.2 \neq 32 \)

Solve for \( n \):  
Get cross products:  
\[
4 \cdot n = 15 \cdot 7
\]

Solve for variable.  
\[
\frac{4n}{4} = \frac{105}{4}
\]

\[ n = 26 \frac{1}{4} \]

#### Section 6.5
**Applications:** On a map, 2 cm represents 9 km.  How many cm would represent 36 km?

1. Assign Variable:  
   \( n \) = number of centimeters

2. Write Ratio in Words:  
   \( \frac{\text{Centimeters}}{\text{Kilometers}} \)

3. Write Ratio of Given Values:  
   \( \frac{2 \text{ cm}}{9 \text{ km}} \)

4. Write Ratio with Variable:  
   \( \frac{n \text{ cm}}{36 \text{ km}} \)

5. Write Proportion:  
   \( \frac{2 \text{ cm}}{9 \text{ km}} = \frac{n \text{ cm}}{36 \text{ km}} \)

6. Solve Proportion:  
   \( \frac{2}{9} = \frac{n}{36} \)

On the map, 8 cm will represent 36 km.
Chapter Review

Simplify the ratio.
1. \[
\frac{12}{45}
\]
2. 35 to 63
3. 36:45

Write each in simplest fraction form.
4. 12 grams to 15 milliliters
5. 85 miles to 15 gallons
6. 100 pages in 30 hours
7. 124 calories in 6 servings

Determine the unit rate.
8. 62 calories per 4 servings
9. 90 miles in 4 hours
10. $345 for 3 months of insurance
11. 20 pages in 8 hours

Determine which purchase is the better deal.
12. 2 boxes for $3.69 or 3 boxes for $5.99
13. 24 cans for $5.99 or 10 cans for $2.70
14. $600 for 120 gallons or $450 for 60 gallons
15. 23 surf boards for $155.25 or $93.75 for 15 surf boards
16. 12 towels for $47.88 or 8 towels for $31.60

Determine if the proportion is true or false.
17. \[
\frac{11}{12} = \frac{33}{39}
\]
18. \[
\frac{9}{25} = \frac{45}{125}
\]
19. \[
\frac{14}{21} = \frac{18}{27}
\]
20. \[
\frac{35}{42} = \frac{50}{72}
\]

Solve the following proportions.
21. \[
\frac{5}{9} = \frac{n}{63}
\]
22. \[
\frac{3}{n} = \frac{12}{16}
\]
23. \[
\frac{7}{9} = \frac{28}{x}
\]
24. \[
\frac{32}{48} = \frac{4}{w}
\]
25. \[
\frac{7}{n} = \frac{15}{20}
\]
26. \[
\frac{6.2}{2} = \frac{43.4}{x}
\]
27. \[
\frac{4}{n} = \frac{2}{\frac{1}{8}}
\]
28. \[
\frac{2}{\frac{7}{3}} = \frac{9}{m}
\]
Translate each into a proportion and solve the problem.

29. Tony gets 9 miles per gallon of gasoline in his truck. If he has 11 gallons of gasoline in his truck, how far can he go?

30. On a map, 2 inches represent 7 miles. How many inches will represent a distance of 91 miles?

31. A recipe calls for $\frac{1}{2}$ teaspoon of vanilla for every 2 teaspoons of baking powder. How much vanilla will be used with 7 teaspoons of baking powder?

32. Brian drives 360 miles on an 18 gallon tank of gasoline. If the car is showing 35 miles until empty, how many gallons of gas are in the car?

33. Jennifer sells five pieces of her homemade jewelry, earning $31.50 profit. How many pieces does she need to sell in order to earn $107.10 profit?

34. John and Joe bought 10 dozen steamed crabs for $85. If Bill purchased 3 dozen crabs from the same company at the same rate, how much does he pay?

35. Jaime had to write a twenty page paper for school. If it took her seven hours to write four pages, how long will it take her to write the entire paper?

**Mixed Review**

36. Convert 64 fluid ounces to quarts.

37. Convert 0.002 hm to cm.

38. Compute $(6 - 8)^3 + |7 - 10| - 36 \div \sqrt{36}$

39. Write $\frac{1}{8}$ as a decimal with three decimal places.

40. Compute $\frac{1}{6} - \frac{3}{4}$

41. Compute $-3 \cdot \frac{3}{5} \cdot \frac{2}{3}$

42. Mary was training for a marathon. Over a week, she tracked the distance she ran each day to be 18.5 miles, 20.5 miles, 19 miles, 21.5 miles, 22 miles, and 21.5 miles. What are the mean, median, and mode for the distances she ran that week?

43. Determine the area of a circle with radius of 12 inches. Use $\pi = 3.14$. 
44. Determine the circumference of a circle with diameter of 84 cm. Use \( \pi = \frac{22}{7} \).

45. Distribute \(-3(8x - 5)\).

46. Solve \( x + 18 = -18 \)

47. Solve \( \frac{x}{4} = 2 \)

48. Solve \( 6x - 2x = 8 \)

49. Solve \( \frac{2}{3}x - \frac{1}{4} = 5 \)

50. Three-tenths of a number plus 6.2 is 3.8. Translate into a math equation and solve the equation to determine the number.
Chapter 6 Review Answers

1. \( \frac{4}{15} \)
2. 5 to 9
3. 4:5
4. \( \frac{4 \text{ grams}}{5 \text{ milliliters}} \)
5. \( \frac{17 \text{ miles}}{3 \text{ gallons}} \)
6. \( \frac{10 \text{ pages}}{3 \text{ hours}} \)
7. \( \frac{62 \text{ calories}}{3 \text{ servings}} \)
8. 15.5 calories per serving
9. 22.5 miles per hour
10. $115 per month
11. 2.5 pages per hour
12. 2 boxes for $3.69
13. 24 cans for $5.99
14. $600 for 120 gallons
15. $93.75 for 15 surf boards
16. 8 towels for $31.60
17. False
18. True
19. True
20. False
21. \( n = 35 \)
22. \( n = 4 \)
23. \( x = 36 \)
24. \( w = 6 \)
25. \( n = \frac{28}{3} = 9 \frac{1}{3} \)
26. \( x = 14 \)
27. \( n = \frac{3}{16} \)
28. \( m = \frac{14}{15} \)
29. 99 miles
30. 26 inches
31. \( \frac{7}{4} = 1 \frac{3}{4} \) tsp of vanilla
32. 1.75 gallons or \( \frac{3}{4} \) gallons
33. 17 pieces of jewelry
34. $25.50
35. 35 hours
36. 2 quarts
37. 20 cm
38. – 11
39. 0.125
40. \( -\frac{7}{12} \)
41. – 24
42. Mean = 20.5 miles
   Median = 21 miles
   Mode = 21.5 miles
43. 452.16 in\(^2\)
44. 264 cm
45. \(-24x + 15\)
46. \(x = -36\)
47. \(x = 8\)
48. \(x = 2\)
49. \(x = \frac{63}{8} = 7 \frac{7}{8}\)
50. – 8
1. Simplify $7x - 9x + 10 - 4$

2. Simplify $-4y + 9x + 3x - 11y + 8y - 6x$

3. Simplify $\frac{3}{4}x - \frac{5}{4}x - \frac{2}{5} - \frac{1}{3}$

4. Evaluate $7x - 3y$ if $x = 4$ and $y = -2$.

5. Evaluate $x^3$ if $x = -2$.

6. Distribute $4(p - 6)$

7. Distribute $-\frac{4}{7}(7h - 35)$

8. Fill in the blanks: An example of the __________ property of ________ is $9 + y = y + 9$.

9. Rewrite $4 \cdot (y \cdot c)$ using the associative property of multiplication

10. Determine whether $v = 8$ is a solution of $7 = v - 15$

11. Solve $9 + x = 7$

12. Solve $y - 7.4 = 9.3$

13. Solve $5x = 55$

14. Solve $\frac{z}{6} = -2$

15. Solve $5y + 4 = 18$

16. $6(2x + 3) - 10x = 4$
17. The sum of 6 and three times a number is 11. Determine the number.

18. Two fifths of a number is six sevenths. Determine the number.

19. Write the following ratio using a colon and as a fraction. 6 to 17

20. Simplify the ratio: 16:28

21. A shelter has 27 cats and 21 dogs. What is the ratio of dogs to cats?

22. Write the ratio for $75 per 12 hours in simplest fraction form.

23. Determine the unit rate. 84 grams of sugar per 14 ounces of cereal.

24. A jogger can run 17.5 miles in 4 hours. What is the unit rate in miles per hour?

25. Determine whether the following proportion is true or false. \( \frac{16}{7} = \frac{30}{13} \)

26. Solve the proportion. Give the answer in decimal form. \( \frac{2.1}{1.4} = \frac{5.4}{n} \)

27. Solve the proportion. Give the answer in fraction form. \( \frac{\frac{3}{4}}{n} = \frac{6}{\frac{20}{9}} \)

28. Translate into a proportion and solve the problem. 15 mg of a drug are to be given every 5 hours. How much of the drug should be given in 24 hours?

29. Translate into a proportion and solve the problem. There are 16 salmon in 50 cubic tons of water in a lake. In how many cubic tons of water would you need to find 72 salmon?

30. Translate into a proportion and solve the problem. 2.5 cups of mix make 14 pancakes. How much mix is needed to make 35 pancakes?
### Unit Three Review Answers

1. $-2x + 6$
2. $-7y + 6x$
3. $-\frac{1}{2}x - \frac{11}{15}$
4. 34
5. -8
6. $4p - 24$
7. $-4h + 20$
8. Commutative; addition
9. $(4 \cdot y) \cdot c$
10. No
11. $x = -2$
12. $y = 16.7$
13. $x = 11$
14. $z = -12$
15. $y = \frac{14}{5}$
16. $x = -7$
17. $6 + 3x = 11; \frac{5}{3}$
18. $\frac{2}{5}x = \frac{6}{7}; \frac{15}{7}$
19. 6:17, $\frac{6}{17}$
20. 4:7
21. $\frac{7 \text{ dogs}}{9 \text{ cats}}$
22. $\frac{\$25}{4 \text{ hours}}$
23. 6 g of sugar/oz
24. 4.375 miles/hour
25. False
26. $n = 3.6$
27. $n = \frac{5}{18}$
28. 72 mg
29. 225 cubic tons of water
30. 6.25 cups